Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 38 (1992)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: EMMY NOETHER: HIGHLIGHTS OF HER LIFE AND WORK
Autor: Kleiner, Israel
Kapitel: C. Her legacy
DOI: https://doi.org/10.5169/seals-59486

Considerable progress on the structure of division algebras was made in the late 1920s and early 1930s. The Albert-Brauer-Hasse-Noether Theorem was a high point of these researches. It should be stressed, however, that even today much is still unknown about finite-dimensional division algebras.

C. Her legacy

The concepts Emmy Noether introduced, the results she obtained, and the mode of thinking she promoted, have become part of our mathematical culture. As Alexandrov put it ([2], p. 158):

> It was she who taught us to think in terms of simple and general algebraic concepts — homomorphic mappings, groups and rings with operators, ideals — and not in cumbersome algebraic computations; and [she] thereby opened up the path to finding algebraic principles in places where such principles had been obscured by some complicated special situation...

Moreover, as Weyl noted, "her significance for algebra cannot be read entirely from her own papers; she had great stimulating power and many of her suggestions took shape only in the works of her pupils or co-workers" ([41], pp. 129-130). Indeed, Weyl himself acknowledged his indebtedness to her in his work on groups and quantum mechanics. Among others who have *explicitly* mentioned her influence on their algebraic works are Artin, Deuring, Hasse, Jacobson, Krull, and Kurosh.

Another important vehicle for the spread of Emmy Noether's ideas was the now-classic treatise of van der Waerden entitled "Modern Algebra", first published in 1930. (It was based on lectures of Noether and Artin — see [39].) Its wealth of beautiful and powerful ideas, brilliantly presented by van der Waerden, has nurtured a generation of mathematicians. The book's immediate impact is poignantly described by Dieudonné and G. Birkhoff, respectively:

> I was working on my thesis at that time; it was 1930 and I was in Berlin. I still remember the day that van der Waerden came out on sale. My ignorance in algebra was such that nowadays I would be refused admittance to a university. I rushed to those volumes and was stupefied to see the new world which opened before me. At that time my knowledge of algebra went no further than *mathématiques spéciales,* determinants, and a little on the solvability of equations and unicursal curves. I had graduated from the École Normale and I did not know what an ideal was, and only just knew what a group was! This gives you an idea of what a young French mathematician knew in 1930 ([13], p. 137).

> Even in 1929, its concepts and methods [i.e., of "modern algebra"] were still considered to have marginal interest as compared with those of analysis in most universities, including Harvard. By exhibiting their mathematical and philosophical unity and by showing their power as developed by Emmy Noether and her other younger colleagues (most notably E. Artin, R. Brauer, and H. Hasse), van der Waerden made "modern algebra" suddenly seem central in mathematics. It is not too much to say that the freshness and enthusiasm of his exposition electrified the mathematical world — especially mathematicians under 30 like myself ([4], p. 771).

A number of mathematicians and historians of mathematics have spoken of the "algebraization of mathematics" in this century (see e.g. [32]). Witness the *terminological* penetration of algebra into such fields as algebraic geometry, algebraic topology, algebraic number theory, algebraic logic, topological algebra, Banach algebras, von Neumann algebras, Lie groups, and normed rings. Emmy Noether's influence is evident directly in several of these fields and indirectly in others. She, too, seemed to have acknowledged that, when she said in a letter to Hasse in 1931: "My methods are really methods of working and thinking; this is why they have crept in everywhere anonymously" ([12], p. 61). Alexandrov and Hopf confirm this in the preface to their book on topology: "Emmy Noether's general mathematical insights were not confined to her specialty — algebra — but affected anyone who came in touch with her" ([12], p. 61). In fact, they, too (and, more importantly, algebraic topology) were major beneficiaries of her insights. As Jacobson notes ([22], p. v):

> As is quite well known, it was Emmy Noether who persuaded Alexandrov and... Hopf to introduce group theory into combinatorial topology and to formulate the then existing simplicial homology theory in group theoretic terms in place of the more concrete setting of incidence matrices.

Algebraic geometry is another area which witnessed very extensive algebraization beginning in the late 1920s and early 1930s. The testimonies of Zariski and van der Waerden, respectively, two of its foremost practitioners who were deeply involved in this process of algebraization, are revealing:

> It was a pity that my Italian teachers never told me there was such a tremendous development of the algebra which is connected with algebraic geometry. I only discovered this much later, when I came to the United States ([33], pp. 36-37).

> When I came to Göttingen in 1924, a new world opened up before me. I learned from Emmy Noether that the tools by which my questions [in

algebraic geometry] could be handled had already been developed... ([34], p. 32).[1])

Emmy Noether was a visiting professor in Moscow in 1928-1929. Alexandrov described the impact she has had on Pontryagin's work in the theory of continuous groups (topological algebra):

> It is not hard to follow the influence of Emmy Noether on the developing mathematical talent of Pontryagin; the strong algebraic flavour in Pontryagin's work undoubtedly profited greatly from his association with Emmy Noether ([2], p. 175).

I will give the last word to Garrett Birkhoff who, in an article in 1976 describing the rise of abstract algebra from 1936 to 1950, said the following ([5], p. 81):

> If Emmy Noether could have been at the 1950 [International] Congress [of Mathematicians], she would have felt very proud. Her concept of algebra had become central in contemporary mathematics. And it has continued to inspire algebraists ever since.

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[1]) To put this statement in perspective, van der Waerden precedes it with the following comments: "In the beginning of our century, many people felt that the theory of invariants was a mighty tool in algebraic geometry... I soon discovered that the real difficulties of algebraic geometry cannot be overcome by calculating invariants and covariants" ([39, p. 32).