7. Schmidt's Game

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **38 (1992)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **23.07.2024**

Good also obtained the estimate $.5306 < \dim \mathscr{E}_2 < .5320$. This was improved by Bumby [48] in 1985 to $.5312 \leqslant \dim \mathscr{E}_2 \leqslant .5314$. More recently, Hensley [140] showed that $.53128049 < \dim \mathscr{E}_2 < .53128051$. For other results on the Hausdorff dimension of $\mathscr{E}_k$ and related sets, see Jarník [153]; Besicovitch [30]; Rogers [262]; Baker and Schmidt [21]; Hirst [147, 148]; Billingsley and Henningsen [32]; Cusick [63, 64, 65]; Pollington [245]; Kaufman [158]; Marion [202]; Gardner and Mauldin [115]; Ramharter [253, 254]; and Hensley [139, 141, 308, 309].

7. Schmidt's Game

W. M. Schmidt [270] introduced the following two-player game, called an (α, β) game: let α, β be real numbers with $0 < \alpha, \beta < 1$. First Bob chooses a closed interval on the real line, called B_1. Then Alice chooses a closed interval $A_1 \subset B_1$, such that the length of A_1 is α times the length of B_1. Then Bob chooses a closed interval $B_2 \subset A_1$, such that the length of B_2 is β times the length of A_1, and so on. If the intersection of all the intervals A_i is a number with bounded partial quotients, then Alice is declared the winner; otherwise Bob is declared the winner.

Schmidt showed that if $0 < \alpha < 1/2$, then Alice always has a winning strategy for this game. This is somewhat surprising, since as we have seen above, the set $\mathscr{E}$ of numbers with bounded partial quotients has Lebesgue measure 0.

Using the theory of (α, β) games, Schmidt also reproved the result of Jarník that $\mathscr{E}$ has Hausdorff dimension 1.

Several papers have proved other results on (α, β) games: see Schmidt [271]; Freiling [109, 110]; and Dani [70, 71, 72]. Also see Schmidt [272, Chapter 3].

8. Hall's theorem

If S and T are sets, then by $S + T$ we mean the set

$$\{s + t \mid s \in S, t \in T\} .$$

Similarly, by $S \cdot T$ we mean the set

$$\{st \mid s \in S, t \in T\} .$$

If S is a set of Lebesgue measure zero, then it is quite possible for $S + S$ to have positive measure. For example, if C denotes the Cantor set (numbers