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Artikel: DENSITY RESULTS ON FAMILIES OF DIOPHANTINE EQUATIONS WITH FINITELY MANY SOLUTIONS

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According to the theorem of Schinzel and Tijdeman, for every $m \geqslant 2$ there exists $C_m > 0$ (and effectively computable) such that if (x, y, z) is a non-trivial solution of (E_m), then $|x|, y, z < C_m$.

LEMMA 7. *With above hypotheses, if* $m \geqslant 2$ *and* $k > C_m$, *then* (E_{km}) *has only trivial solutions.*

Proof. Let $k > C_m$, let (x, y, z) be a non-trivial solution of (E_{km}). So $f(x^{km}) = y^z$, thus (x^k, y, z) is a non-trivial solution of (E_m). Hence $|x^k| < C_m < k$. Then $|x| \leqslant 1$ and therefore $f(0), f(1)$ or $f(-1)$ is a proper power, which is contrary to the hypothesis. □

Let $F = \{m \geqslant 2 \mid (E_m) \text{ has only trivial solutions}\}$.

PROPOSITION 8. $\mu(F) = 1$.

Proof. Let F' be the complement of the set F; it suffices to show that $\mu(F') = 0$.

For each prime p, $kp \in F$ for each $k > C_p$, according to lemma 7. So $\mathbf{N}p \cap F'$ is finite. By lemma 2, $\mu(F') = 0$. □

Actually, the complement of F is finite, if f has at least two simple zeroes.

We note the following interesting application. Let a, b, c be non-zero integers, such that $-\frac{c}{b}$ and $\pm\frac{a}{b} - \frac{c}{b}$ are not zero, nor 1, nor proper powers. Let $f = \frac{a}{b}X - \frac{c}{b}$. The above result is applicable to the polynomial f and yields:

The set of $m \geqslant 3$ *such that there exist integers* $n \geqslant 2$, *and* x, y, *with* $y \geqslant 2$, *satisfying* $ax^m - by^n = c$, *has uniform density equal to zero.*

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