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AN ERGODIC ADDING MACHINE ON THE CANTOR SET

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ABSTRACT. We calculate all ergodic measures for a specific function F on the unit interval. The supports of these measures consist of periodic orbits of period 2^n and the classical ternary Cantor set. On the Cantor set, F is topologically conjugate to an "adding machine" in base 2. We show that F is representative of the class of functions with zero topological entropy on the unit interval, already analyzed in the literature, and its behavior is therefore typical of that class.

I. INTRODUCTION

The dynamical behavior of the quadratic function $f_c(x) = x^2 - c$ has been extensively studied as the parameter c is varied. For example, $c_0 = 1.401155189...$ is the smallest value of c for which $f_c(x)$ has infinitely many distinct periodic orbits [1-3]. As c approaches this number through smaller values, the dynamical system, $x \to f_c(x)$, progresses through the famous period doubling route to chaos. When $c = c_0$, the dynamical behavior of $f(x) \equiv f_c(x)$ includes the following properties:

1. There is a Cantor set K which is an attractor and $f: K \to K$
2. All periodic points of f have period 2^n for some n.
3. There are periodic points which are arbitrarily close to K.
4. With the restriction of $f(x)$ to an appropriate interval I such that $f(I) \subset I$, there are just two possibilities for the orbit of a point $x_0 \in I$: either $f^k(x_0)$ is in a periodic orbit for some k, or $f^k(x_0)$ converges to K as k increases.
5. The restriction of f to K is topologically equivalent to a function on 2-adic integers which adds 1 to its argument (this "adding machine" will be described in detail below).