

Zeitschrift: L'Enseignement Mathématique
Band: 40 (1994)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: AN ERGODIC ADDING MACHINE ON THE CANTOR SET

Kurzfassung

Autor: Collas, Peter / Klein, David

DOI: <https://doi.org/10.5169/seals-61113>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 06.10.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

AN ERGODIC ADDING MACHINE ON THE CANTOR SET

by Peter COLLAS and David KLEIN

ABSTRACT. We calculate all ergodic measures for a specific function F on the unit interval. The supports of these measures consist of periodic orbits of period 2^n and the classical ternary Cantor set. On the Cantor set, F is topologically conjugate to an “adding machine” in base 2. We show that F is representative of the class of functions with zero topological entropy on the unit interval, already analyzed in the literature, and its behavior is therefore typical of that class.

I. INTRODUCTION

The dynamical behavior of the quadratic function $f_c(x) = x^2 - c$ has been extensively studied as the parameter c is varied. For example, $c_0 = 1.401155189\dots$ is the smallest value of c for which $f_c(x)$ has infinitely many distinct periodic orbits [1-3]. As c approaches this number through smaller values, the dynamical system, $x \rightarrow f_c(x)$, progresses through the famous period doubling route to chaos. When $c = c_0$, the dynamical behavior of $f(x) \equiv f_c(x)$ includes the following properties:

1. There is a Cantor set K which is an attractor and $f : K \rightarrow K$
2. All periodic points of f have period 2^n for some n .
3. There are periodic points which are arbitrarily close to K .
4. With the restriction of $f(x)$ to an appropriate interval I such that $f(I) \subset I$, there are just two possibilities for the orbit of a point $x_0 \in I$: either $f^k(x_0)$ is in a periodic orbit for some k , or $f^k(x_0)$ converges to K as k increases.
5. The restriction of f to K is topologically equivalent to a function on 2-adic integers which adds 1 to its argument (this “adding machine” will be described in detail below).