Finally we like to show that the non-emptiness condition on the index cone of a projective 3-fold with $h^{0,2} = 0$ gives non-trivial restrictions for the possible cup-forms if $b_2 \geqslant 4$. Further investigations of this condition will appear elsewhere [Sch].

EXAMPLE 17. Let H be a free $\mathbf{Z}$-module of rank 4 with basis $(e_i)_{i=1,\dots,4}$. Consider a trilinear form $F \in S^3 H^\vee$ and its adjoint map $F^t : H \to S^2 H^\vee$. The image $F^t(h)$ of an element $h \in H$ is in terms of the chosen basis $(e_i)_{i=1,\dots,4}$ represented by the symmetric 4×4-matrix $[[he_i e_j]]_{i,j=1,\dots,4}$. Suppose this matrix is a diagonal sum $[[he_i e_j]]_{i,j=1,2} \oplus [[he_k e_l]]_{k,l=3,4}$ such that the determinants of both 2×2-matrices are negative for every $h \in H \setminus \{0\}$.

In this case $F^t(h)$ were of signature $(1, -1, 1, -1)$ for every $h \in H \setminus \{0\}$, and we would have $I_F = \mathscr{H}_F = \emptyset$.

All these conditions can be met, e.g. by setting $e_1^2 e_2 = e_2^3 = e_3^2 e_4 = e_4^3 = 1$, $e_1 e_2^2 = e_3 e_4^2 = 2$, and $e_i e_j e_k = 0$ otherwise. In this particular case the image of $h = \sum_{i=1}^4 h_i e_i$ under F^t is represented by the matrix

$$\left[\begin{array}{cc|cc} h_2 & h_1 + 2h_2 & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ h_1 + 2h_2 & 2h_1 + h_2 & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{0}} & h_4 & h_3 + 2h_4 \\ & & h_3 + 2h_4 & 2h_3 + h_4 \end{array}\right],$$

which has a positive determinant unless $h = 0$.

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Ch. Okonek

Institut für Mathematik
Universität Zürich
Winterthurer Strasse 190
8001 Zürich
Switzerland

A. Van de Ven

Department of Mathematics
Leiden University
Niels Bohrweg 1
Postbus 9512
2300 RA Leiden