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Autor: Wingren, Peter

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Now consider the restrictions of $\tilde{f}(x)$ to all $(p+2)$ -generation bands in D_L and D_R , and use the translation properties for G and its derivative g . Then by applying Lemma 2 with $\|g\| = 2^{-2^{p+2}}$, $d > 2^{-2^{p+1}-(p+2)}$, $m(I) = 2^{-n}$ we obtain

$$(23) \quad \mu(D_L \cap Q) + \mu(D_R \cap Q) \leq \left(1 + \text{int} \frac{2^{-n}}{2^{-2^{p+1}-(p+2)}}\right) \cdot 2^{-2^{p+2}}.$$

The number of bands from the $(p+1)$ generation contained in D_0 are $2^{-n}/2^{-2^{p+1}}$, and, since $2^p < n$ by (18), we have, for $\alpha < 2$,

$$(24) \quad \begin{aligned} \mu(Q) = \mu(B_0 \cap Q) &\leq \frac{2^{-n}}{2^{-2^{p+1}}} \cdot \left(1 + \text{int} \frac{2^{-n}}{2^{-2^{p+1}-(p+2)}}\right) \cdot 2^{-2^{p+2}} \\ &\leq 2^{-n} \cdot 2^{-2^{p+1}} + 2^{-2n+p+2} \leq (2^{-n})^2 \cdot (1 + 2^{p+2}) \\ &\leq (2^{-n})^2 (1 + 4n) \leq (2^{-n})^\alpha = |Q|^\alpha \end{aligned}$$

if $1 + 4n \leq 2^{n(2-\alpha)}$.

The Mass Distribution Principle now gives (17) and the proof is complete.

Remark. The nowhere-differentiability of the constructed function f is omitted in the statement of the Theorem. However this property can be established by minor changes to the proof in [RHA] or the proof of Theorem 2-9 in [D-W]. The continuity of $f(x)$ follows from uniform convergence of the series (4).

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Peter Wingren

Umeå University
Department of Mathematics
S-901 87 Umeå (Sweden)