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CHAOTIC GROUP ACTIONS

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ABSTRACT. We introduce the notion of chaotic group actions and give a preliminary report on their properties. In particular, we show that a group G possesses a faithful chaotic action on some Hausdorff space if and only if G is residually finite. This gives an elementary unified proof of the residual finiteness of certain groups. We also show that the circle does not admit a chaotic action of any group, whilst every smooth compact surface admits a chaotic $\mathbf{Z}$-action.

1. INTRODUCTION

In recent years an enormous amount of work has been conducted on chaotic dynamical systems. Most of this work has been concerned with the iteration of single maps; in other words, with group (or semi-group) actions of the additive group $\mathbf{Z}$. Now, according to R. Devaney's [D2] definition (see also [BBCDS], [GW] and [Si]), a map is chaotic if it is topologically transitive and if the set of periodic points is dense. The purpose of this present paper is to introduce the analogous notion for actions of arbitrary groups:

Definition. Suppose that a group G acts continuously on a Hausdorff topological space M. Then we say that the action of G on M is *chaotic* if the following two conditions are met:

(a) *topological transitivity:* for every pair of non-empty open subsets U and V of M, there is an element $g \in G$ such that $g(U) \cap V \neq \varnothing$.

(b) *finite orbits dense:* the set of points in M whose orbit under G is finite is a dense subset of M.

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