

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 41 (1995)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: HIGHER EULER CHARACTERISTICS (I)
Autor: Geoghegan, Ross / Nicas, Andrew
Kurzfassung
DOI: <https://doi.org/10.5169/seals-61816>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 18.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

HIGHER EULER CHARACTERISTICS (I)

by Ross GEOGHEGAN¹⁾ and Andrew NICAS²⁾

To Peter Hilton on the occasion of his 70-th birthday.

ABSTRACT. The classical Euler characteristic $\chi \equiv \chi_0$ of a finite complex lies at the bottom of a sequence of homotopy invariants. The next invariant in this sequence χ_1 is introduced here and studied in some detail. The rest of the sequence, χ_n with $n \geq 2$, will be discussed in a sequel paper. Applications to geometric group theory are found by considering the behavior of χ_1 on an aspherical finite complex of fundamental group G . Just as the $\chi(G) \neq 0$ implies that the center of G is trivial (Gottlieb's Theorem), it is shown here that (under a weak additional hypothesis and using rational coefficients) $\chi_1(G) \neq 0$ implies that the center of G is infinite cyclic. We also find a generalization of Gottlieb's Theorem in which the Lefschetz number of an automorphism of G is related to the fixed subgroup of the automorphism.

INTRODUCTION

From our point of view, the classical Euler characteristic of a finite complex is “zero-th order”. In this paper we introduce a “first order” analog, a new invariant in topology and group theory. In a sequel paper and in [GNO] we extend these ideas to an “ n -th order” Euler characteristic for all positive n .

For a finite complex X , the new invariant $\chi_1(X; R)$, defined in §1, comes in different forms, depending on the coefficient ring R ; and a more sophisticated version $\tilde{\chi}_1(X; R)$ defined in §2, involves the universal cover of X . By contrast, the classical analogs of these are essentially the same, namely the integer $\chi(X)$. We should tell the reader from the start that all our first order invariants are trivial if X is simply connected.

1991 *Mathematics Subject Classification*. Primary 55M20; Secondary 19D55, 20F28, 20F32.

¹⁾ Partially supported by the National Science Foundation.

²⁾ Partially supported by the Natural Sciences and Engineering Research Council of Canada.

The authors acknowledge the hospitality of the Institute for Advanced Study and support from NSF grant DMS 9304580.