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## JACOBI SUMS AND STICKELBERGER'S CONGRUENCE

by Keith CONRAD<sup>1</sup>

**ABSTRACT.** We present an extension of a classical congruence for Jacobi sums of two characters to a congruence for arbitrary Jacobi sums. This congruence is used to provide what seems to be a new proof of Stickelberger's congruence for Gauss sums, as well as a new explanation for the appearance of base  $p$  digits in Stickelberger's congruence. It is also shown that in fact the Jacobi sum congruence and Stickelberger's congruence are equivalent.

### INTRODUCTION

About a century ago, Stickelberger established a congruence for Gauss sums over a finite field which has had useful implications for the study of cyclotomic fields. A generalized version of a classical congruence for Jacobi sums of two characters will be proven which is ultimately shown to be equivalent to Stickelberger's congruence. In particular, this allows for a new proof of Stickelberger's congruence and a new explanation for the form of the congruence.

Before discussing finite fields, we will need to fix a way of representing these fields and the multiplicative characters on them. Let  $p$  be a positive prime,  $q = p^f$  for  $f$  in  $\mathbf{Z}^+$ . We have the following diagram of number fields and primes, where  $\mathfrak{P}_i$  lies over  $\mathfrak{p}_i$ ,  $g = \varphi(q - 1)/f$ , and  $\zeta_p, \zeta_{q-1} \in \mathbf{C}$  denote roots of unity with respective orders  $p$  and  $q - 1$ :

$$\begin{array}{ccc}
 \mathbf{Q}(\zeta_{q-1}, \zeta_p) & \mathfrak{P}_1^{p-1} \cdot \dots \cdot \mathfrak{P}_g^{p-1} & \\
 | & | & \\
 \mathbf{Q}(\zeta_{q-1}) & \mathfrak{p}_1 \cdot \dots \cdot \mathfrak{p}_g & \\
 | & | & \\
 \mathbf{Q} & p &
 \end{array}$$

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