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where the maps α, β, γ and δ are induced by inclusion. In order for γ to be a well defined homomorphism it is necessary to check that the relation $w^{-1}hw = h^{\phi}, h \in H$ is a consequence of the relations $t^{-1}ht = h^{\phi}, [a, t^{-1}w] = 1, [t, w] = 1, h \in H, a \in A$. But this follows because $w^{-1}hw = w^{-1}tt^{-1}htt^{-1}w = w^{-1}th^{\phi}t^{-1}w = h^{\phi}$. Now α is injective because A' is an HNN extension of A (see [DD, p. 33] or [Se, p. 9]) and β is injective because of theorem 6.1. So δ is injective and this proves the theorem. □

THEOREM 6.3. *Let*

$$(*) \qquad u_i(t) = 1, \ i \in I$$

be a set of equations over the group A *where the exponent sum of* t *in each* $u_i(t)$ *is zero. Suppose* $w = w(t) \in A * \langle t \rangle - A$ *and the factors of* w *are all torsion free. Then the set of equations*

$$(**) \qquad u_i(w(t)) = 1, \ i \in I$$

has a solution over A *if and only if the set* $(*)$ *has a solution over* A.

Proof. Let $w(t) = at$ where $a \in A$ has infinite order. Then a solution x for $u_i(w(t)) = 1$ defines a solution at for $(*)$.

Conversely suppose $x \in A'$ is a solution of the set of equations $\{u_i(t) = 1 \mid i \in I\}$. Let G be the subgroup of A' generated by

$$\{x^{-n}ax^n \mid a \in A, n \in \mathbf{Z}\}.$$

Then A is a subgroup of G and G is a subgroup of

$$H = \langle G, t \mid w^{-1}gw = g^{\phi}, g \in G \rangle$$

where $g^{\phi} = x^{-1}gx$ by theorem 6.2. Because of the exponent sum condition $u_i(w) = 1, i \in I$. □

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