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**Autor:** Leuzinger, Enrico  
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## 1. THE FORMULA OF ALLENDOERFER AND WEIL

A  $C^\infty$  (resp.  $C^\omega$ ) manifold with corners is a topological Hausdorff space locally modeled upon a product of lines and half-lines and such that coordinate changes are of class  $C^\infty$  (resp.  $C^\omega$ ). For precise definitions and basic information about this concept we refer to [DH]. A *Riemannian polyhedron* is a compact manifold with corners equipped with a Riemannian metric.

Let  $\mathcal{P}^n$  be an  $n$ -dimensional Riemannian polyhedron with boundary consisting of a finite family of lower dimensional subpolyhedra

$$\mathcal{P}_E^{n-k} \quad (0 \leq k \leq n-1)$$

and with Riemannian metric induced from  $\mathcal{P}^n$ . The *outer angle*  $O(p)$  at a point  $p$  of  $\mathcal{P}_E^{n-k}$  is defined as the set of all unit tangent vectors  $v \in T_p\mathcal{P}^n$  such that  $\langle v, w \rangle_p \leq 0$  for all  $w$  in the tangent cone of  $\mathcal{P}^n$  at  $p$ . Note that  $O(p)$  is a spherical cell bounded by “great spheres” in the  $(k-1)$ -dimensional unit sphere of the normal space of  $\mathcal{P}_E^{n-k} \subset \mathcal{P}^n$  at  $p$ . In [AW] Allendoerfer and Weil define a certain real valued function  $\Psi_{E,k}$  on the outer angles of  $\mathcal{P}_E^{n-k}$ . The explicit form of this function will not be needed in this paper. We shall only use the fact that  $\Psi_{E,k}$  is locally computable from the components of the metric and the curvature tensor of  $\mathcal{P}^n$  and from the components of the second fundamental forms  $\Pi_Z(p), Z \in O(p)$ , of  $\mathcal{P}_E^{n-k}$  in  $\mathcal{P}^n$ . Let  $\Psi dv$  denote the Gauss-Bonnet-Chern form on  $\mathcal{P}^n$  and  $dv_E$  (resp.  $d\omega_{k-1}$ ) the volume element of  $\mathcal{P}_E^k$  (resp. of the standard unit sphere  $S^{k-1}$ ). The *inner Euler characteristic*  $\chi'$  of  $\mathcal{P}^n$  is by definition the Euler characteristic of the open complex consisting of all inner cells in an arbitrary simplicial subdivision of  $\mathcal{P}^n$ .

We can now state the generalized Gauss-Bonnet formula of Allendoerfer-Weil for Riemannian polyhedra (see [AW]).

PROPOSITION 1.1. *Let  $\mathcal{P}^n$  be a Riemannian polyhedron with boundary consisting of a finite family of subpolyhedra  $\mathcal{P}_E^{n-k}$ . Then the inner Euler characteristic of  $\mathcal{P}^n$  is given by*

$$(-1)^n \chi'(\mathcal{P}^n) = \int_{\mathcal{P}^n} \Psi dv + \sum_{k=1}^n \sum_E \int_{\mathcal{P}_E^{n-k}} \left( \int_{O(p)} \Psi_{E,k} d\omega_{k-1} \right) dv_E(p).$$