Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 42 (1996)

Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SQUARE-FREE TOWER OF HANOI SEQUENCES

Autor: HINZ, Andreas M.

Bibliographie

DOI: https://doi.org/10.5169/seals-87878

over the three-letter alphabet $\{\alpha, \beta, \gamma\}$. This is, of course, the smallest possible alphabet with an infinite square–free string (clearly, a square-free word over a two-letter alphabet will come to an end after three entries) with which the whole theory started in the work of Axel Thue [19, Satz 3], [20, Sätze 6, 7, 20].

Obviously, t (as in fact any word with more than 7 elements over a three-letter alphabet) is not strongly square-free. Maybe TH sequences hold a clue for a more direct approach to the question (cf. [6]), if there is an infinite strongly square-free string over a four-letter alphabet, which has been answered positively by V. Keränen [16] employing a computer-aided proof. (An abelian square of length $2 \cdot 6$ in h starts after position 6.)

ACKNOWLEDGEMENTS. I gratefully acknowledge the liberty to work I enjoyed during my visit as a Research Fellow at King's College London, where this note was finished, made possible by EPSRC grant N° GR/K10362. I am indebted to Professor E. B. Davies FRS for his support and hospitality.

Thanks go to the referee for some valuable comments and in particular for bringing reference [16] to my notice.

REFERENCES

[1] AFRIAT, S. N. *The Ring of Linked Rings.* Duckworth (London), 1982.

[2] ALLOUCHE, J.-P., D. ASTOORIAN, J. RANDALL and J. SHALLIT. Morphisms, Squarefree Strings, and the Tower of Hanoi Puzzle. *Amer. Math. Monthly 101* (1994), 651–658.

[3] ALLOUCHE, J.-P. et F. DRESS. Tours de Hanoï et automates. *RAIRO Inform. Théor. Appl. 24* (1990), 1–15.

[4] BERSTEL, J. Some recent results on squarefree words. In *STACS 84 (Lecture Notes in Comput. Sci. 166),* G. Goos, J. Hartmanis (eds.), Springer (Berlin), 1984, 14–25.

[5] BOYER, C. B. *A History of Mathematics.* Princeton University Press (Princeton), 1985.

[6] BROWN, T. C. Is there a sequence on four symbols in which no two adjacent segments are permutations of one another? *Amer. Math. Monthly 78* (1971), 886–888.

[7] CLAUS, N. (= E. LUCAS). *La Tour d'Hanoï, Véritable casse-tête annamite.* P. Bousrez (Tours), 1883.

[8] CLAUS, N. (= E. LUCAS). La Tour d'Hanoï, Jeu de calcul. *Science et Nature, Vol. I, N° 8* (1884), 127–128.

[9] CUMMINGS, L. J. Gray codes and strongly square-free strings. In *Sequences II, Methods in Communication, Security, and Computer Science,* R. Capocelli, A. De Santis, U. Vaccaro (eds.), Springer (New York), 1993, 439–446.

[10] GLAISHER, J. W. L. On the residue of a binomial-theorem coefficient with respect to a prime modulus. *Quart. J. Pure Appl. Math. 30* (1899), 150–156.

[11] DE GUZMÁN, M. The role of games and puzzles in the popularization of mathematics. *Enseign. Math. (2) 36* (1990), 359–368.

[12] HARKIN, D. On the mathematical work of François-Édouard-Anatole Lucas. *Enseign. Math. (2) 3* (1957), 276–288.

[13] HINZ, A.M. The tower of Hanoi, *Enseign. Math. (2) 35* (1989), 289–321.

[14] —— Pascal's triangle and the tower of Hanoi. *Amer. Math. Monthly 99* (1992), 538–544.

[15] HINZ, A. M. and A. SCHIEF. The average distance on the Sierpiński gasket. *Probab. Theory Related Fields 87* (1990), 129–138.

[16] KERÄNEN, V. Abelian Squares are avoidable on 4 letters. In *Automata, Languages and Programming,* W. Kuich (ed.), Springer (Berlin), 1992, 41–52.

[17] PLEASANTS, P. A. B. Non-repetitive sequences. *Proc. Camb. Phil. Soc. 68* (1970), 267–274.

[18] STEWART, I. Four encounters with Sierpiński's gasket. *Math. Intelligencer 17, N° 1* (1995), 52–64.

[19] THUE, A. Über unendliche Zeichenreihen. *Kra. Vidensk. Selsk. Skrifter. I. Mat. Nat. Kl.* 1906 *N° 7*, 1–22 = *Selected Mathematical Papers of Axel Thue,* T. Nagell, A. Selberg, S. Selberg, K. Thalberg (eds.), Universitetsforlaget (Oslo), 1977, 139–158.

[20] —— Ueber die gegenseitige Lage gleicher Teile gewisser Zeichenreihen. *Kra. Vidensk. Selsk. Skrifter. I. Mat. Nat. Kl.* 1912 *N° 1*, 1–67 = *Selected Mathematical Papers of Axel Thue,* T. Nagell, A. Selberg, S. Selberg, K. Thalberg (eds.), Universitetsforlaget (Oslo), 1977, 413–477.

(Reçu le 22 juillet 1995; version révisée reçue le 7 décembre 1995)

Andreas M. Hinz

Mathematisches Institut
Universität München
Theresienstr. 39
D-80333 München
Germany

E-mail: hinz@rz.mathematik.uni-muenchen.de