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over the three-letter alphabet  $\{\alpha, \beta, \gamma\}$ . This is, of course, the smallest possible alphabet with an infinite square-free string (clearly, a square-free word over a two-letter alphabet will come to an end after three entries) with which the whole theory started in the work of Axel Thue [19, Satz 3], [20, Sätze 6, 7, 20].

Obviously,  $t$  (as in fact any word with more than 7 elements over a three-letter alphabet) is not strongly square-free. Maybe TH sequences hold a clue for a more direct approach to the question (cf. [6]), if there is an infinite strongly square-free string over a four-letter alphabet, which has been answered positively by V. Keränen [16] employing a computer-aided proof. (An abelian square of length  $2 \cdot 6$  in  $h$  starts after position 6.)

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