*L'Enseignement Mathématique, t. 43 (1997), p. 207–252*



# AN ALGORITHM FOR CELLULAR MAPS OF CLOSED SURFACES

by Warren DICKS and H. H. GLOVER

ABSTRACT. The purpose of this article is to use diagrammatic methods to give proofs, accessible to algebraists, of some important topological results of H. Kneser, A. L. Edmonds, and R. Skora; we then describe some consequences for homomorphisms between surface groups. Cellular maps between two-dimensional CW-complexes can be represented by diagrams which, in turn, can be interpreted algebraically in terms of fundamental groupoids. For diagrams representing cellular maps between closed surfaces, we show how to apply certain homotopy equivalences algorithmically to obtain a normal-form map, which is a branched covering, or a pinching followed by a covering, or a map which collapses a graph of punctured spheres to a graph immersed in the one-skeleton of the target surface. We then indicate how the algorithm can be expressed entirely in terms of formal manipulations with presentations of surface groupoids, yielding algebraic proofs of results about homomorphisms between surface groups.

## 1. INTRODUCTION

We begin by recalling some basic concepts.

1.1. DEFINITIONS. Let $\beta$ be a map between closed surfaces (without boundary).

Then $\beta$ is a *branched covering* if deleting finitely many points from the source and from the target yields a covering.

We say that $\beta$ is a (possibly trivial) *pinching* if it is obtained by collapsing, to a point, a compact subsurface with a single boundary component.

The (geometric) *degree* of $\beta$, denoted $\mathcal{G}(\beta)$, is the least non-negative integer $d$ such that there is a map $\beta'$ homotopic to $\beta$, such that the inverse image under $\beta'$ of some 2-disk consists of $d$ 2-discs, each mapped homeomorphically by $\beta'$ to the chosen disk.