# A. Appendix

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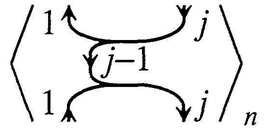
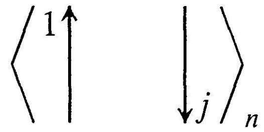
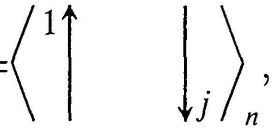
$$= \left([n-j+1] - q^{1/2}[n-j] - q^{-1/2}[n-j] + [n-j-1]\right)$$ 

$$+$$ 

$$=$$  ,

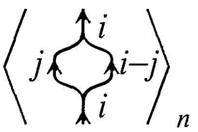
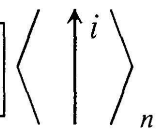
where we use Lemma 5.2 in the third equality. Now the proof for the Reidemeister move II is complete.

*The invariance under the Reidemeister move III.* This is proved by repeated application of Lemma 5.3 and details are omitted. See the proof of Theorem 3.1.

## A. Appendix

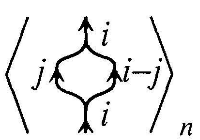
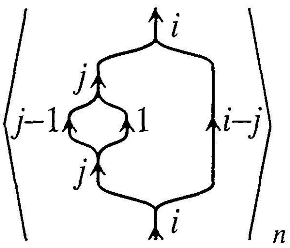
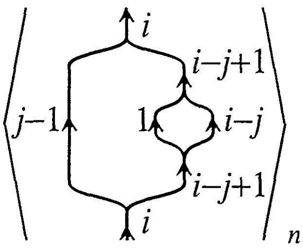
In this appendix, we give proofs of lemmas used in the previous section.

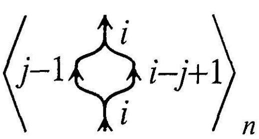
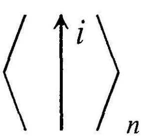
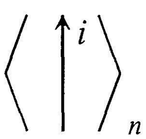
LEMMA A.1.

 $$= \begin{bmatrix} i \\ j \end{bmatrix}$$ 

*for* $i \geq j \geq 0$.

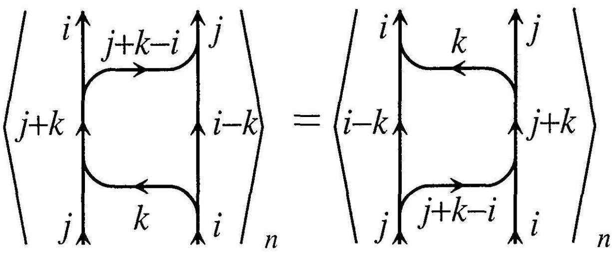
*Proof.* The proof for $j = 1$ is similar to that of Lemma 2.2 and omitted. For $j > 1$ we have

 $$= \frac{1}{[j]}$$  $$= \frac{1}{[j]}$$ 

$$= \frac{[i-j+1]}{[j]}$$  $$= \frac{[i-j+1]}{[j]} \begin{bmatrix} i \\ j-1 \end{bmatrix}$$  $$= \begin{bmatrix} i \\ j \end{bmatrix}$$  ,

where the second equality follows from Lemma 2.6 and the fourth by induction. The proof is complete. □

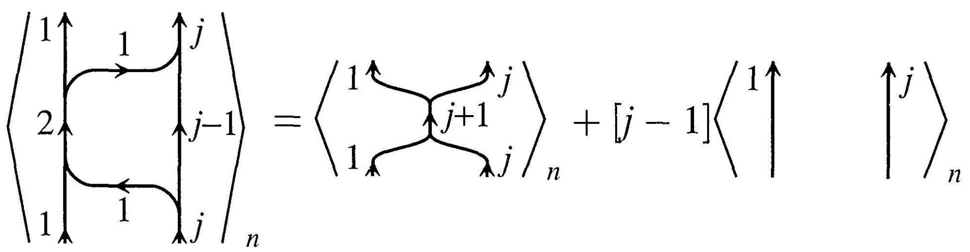
LEMMA A.2.



*for* $j+k \geq i \geq k \geq 0$.

*Proof.* By a $\pi$-rotation and orientation reversing, we get the right hand side from the left hand side. So there is a one to one correspondence between the states of both hand sides and the equality follows from the definition. □

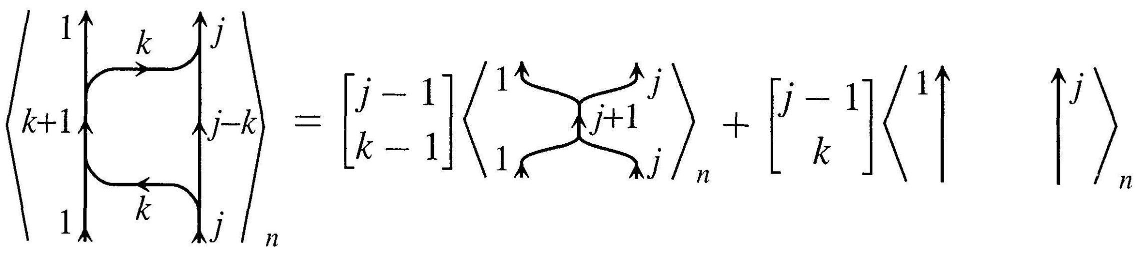
LEMMA A.3.



*for* $j \geq 1$.

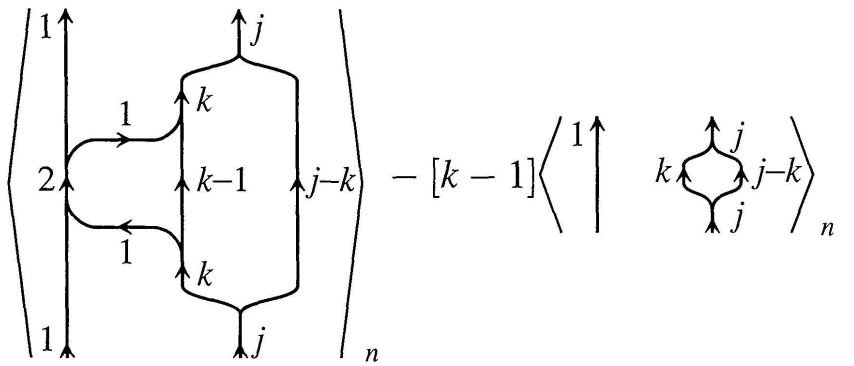
*Proof.* We can prove the equality directly from the definition as in the proofs in the lemmas in §2. Details are omitted. □

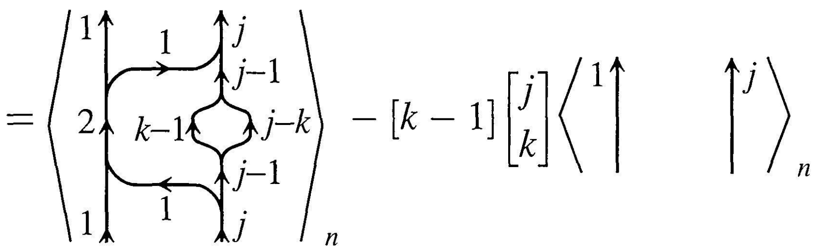
LEMMA A.4.

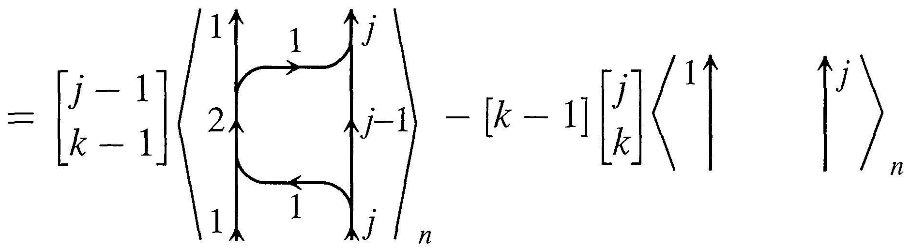


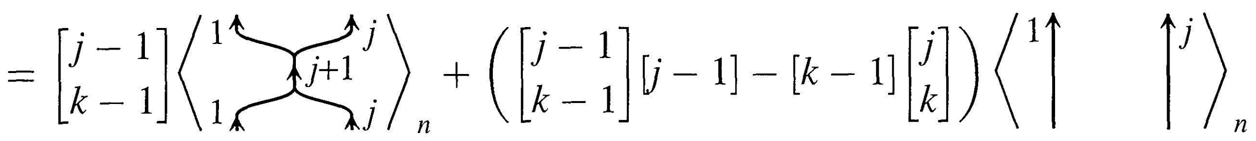
*for* $j \geq k \geq 1$.

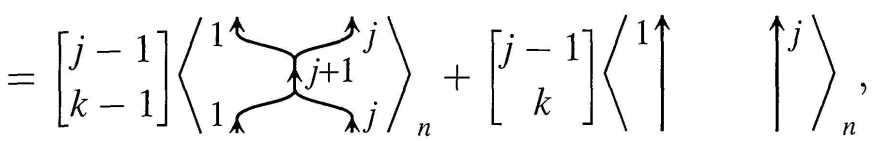
*Proof.* From Lemma A.3 (substituting $j$ with $k$) the left hand side becomes



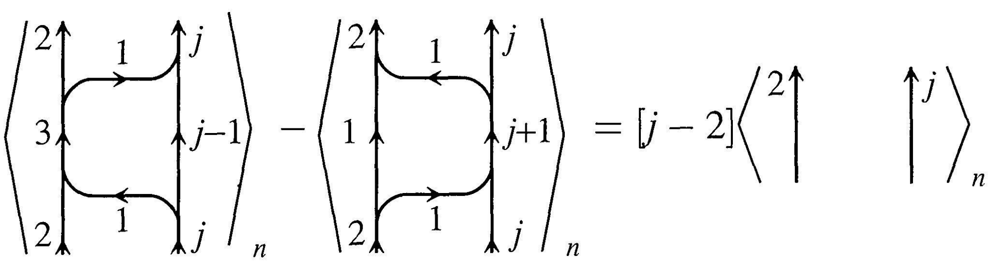



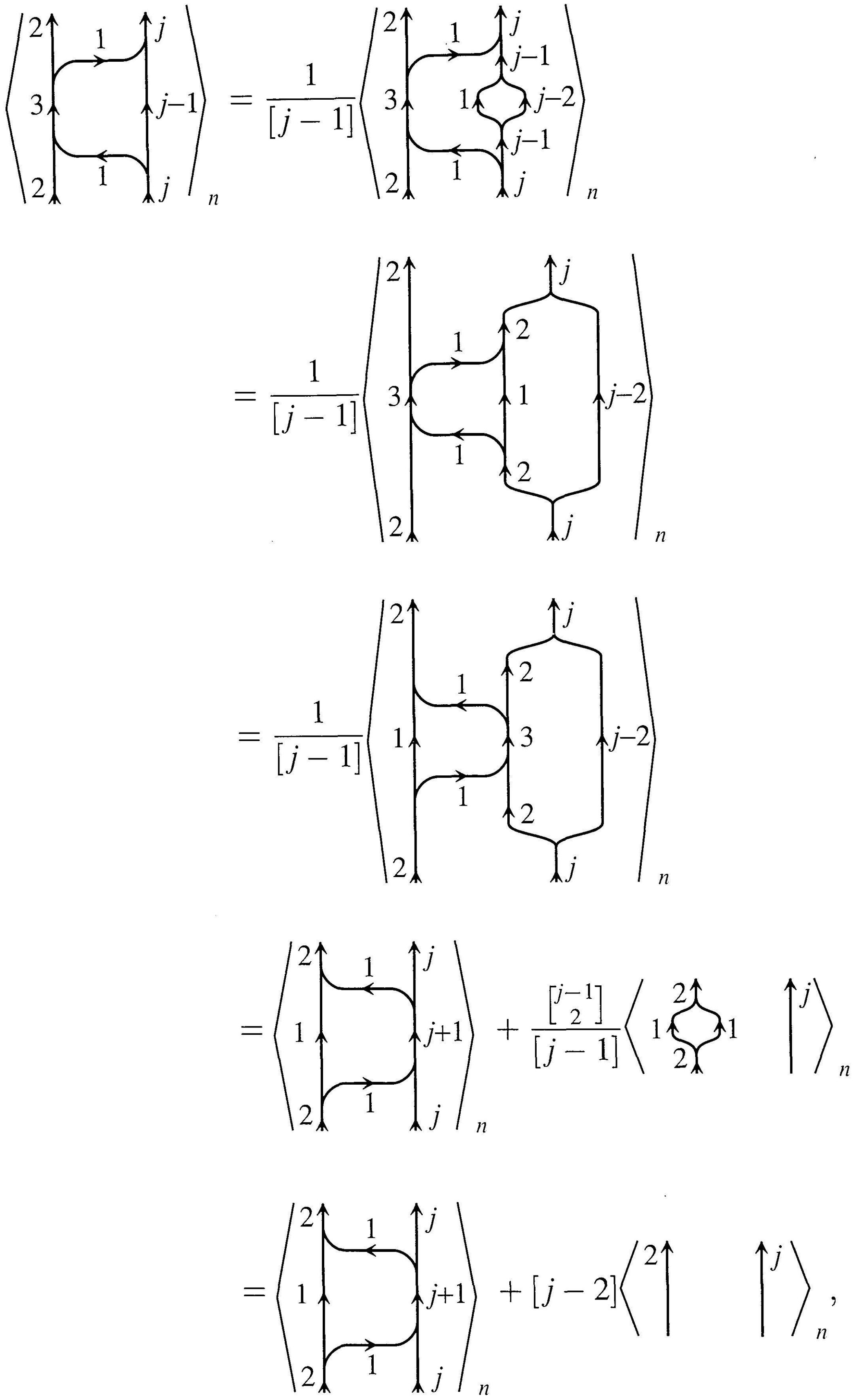

where the third equality follows from Lemma A.3. The proof is complete. □

LEMMA A.5.



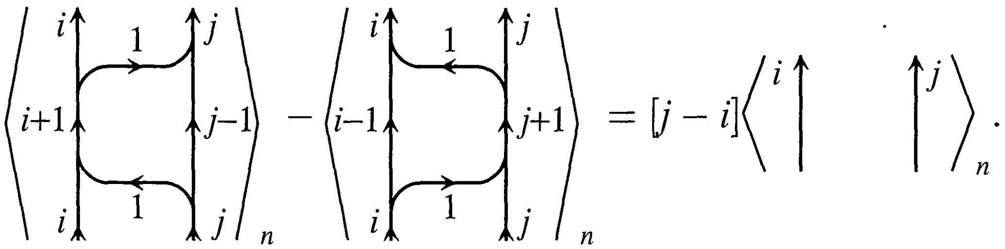

*for* $j \geq 2$.

*Proof.* We have

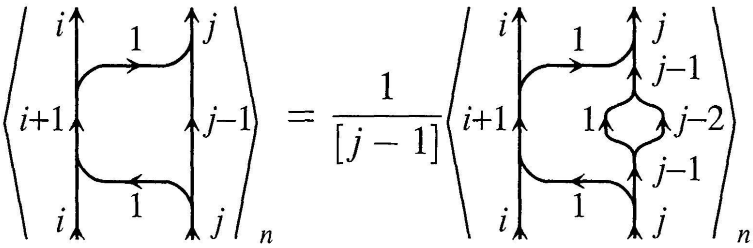


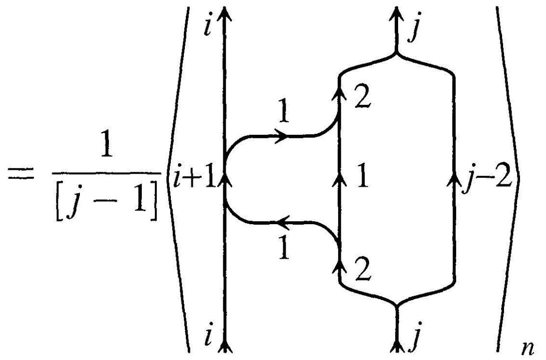

where we use Lemma A.2 in the third equality and Lemma A.4 in the fourth equality. The proof is complete. □

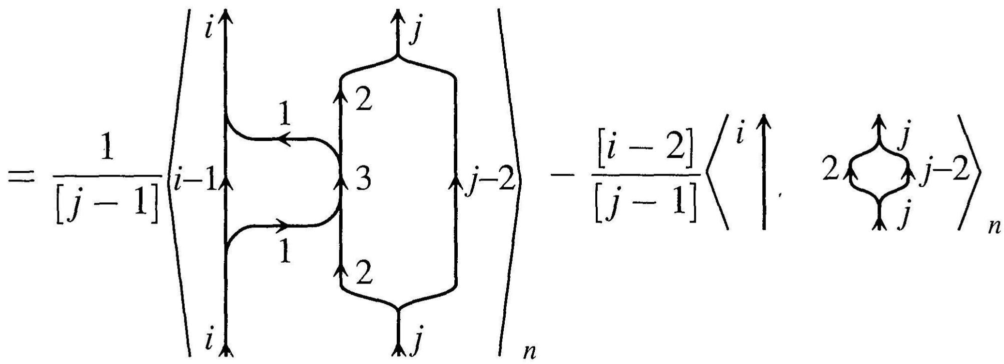
LEMMA A.6.

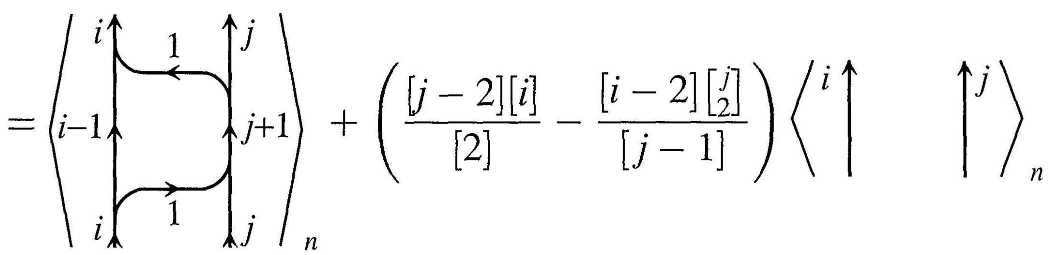


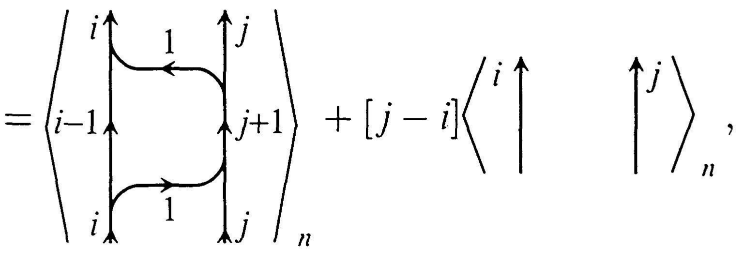

*Proof.* We have



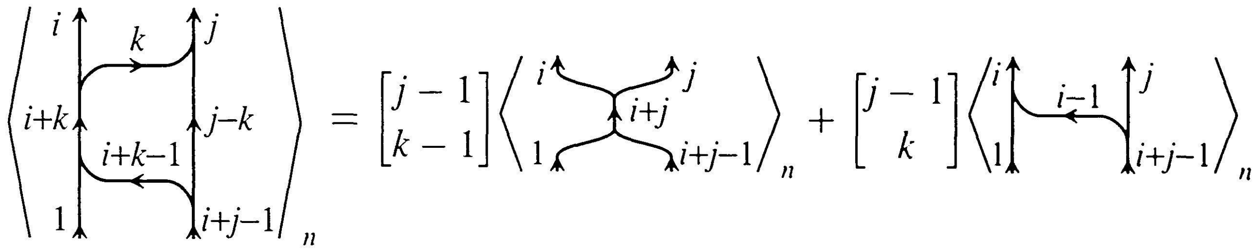





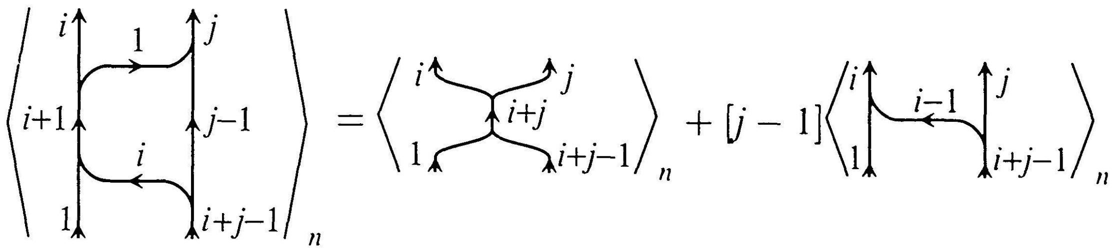
where we use Lemmas A.3 and A.4 in the third and the fourth equalities respectively. The proof is complete. □

LEMMA A.7.



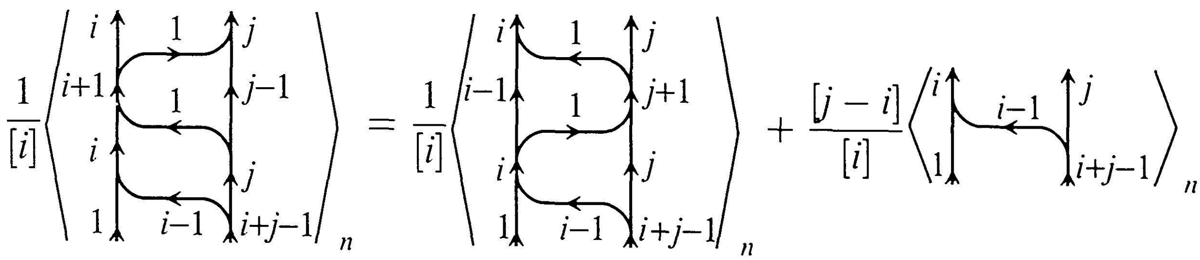
*for* $j > k \geq 1$.

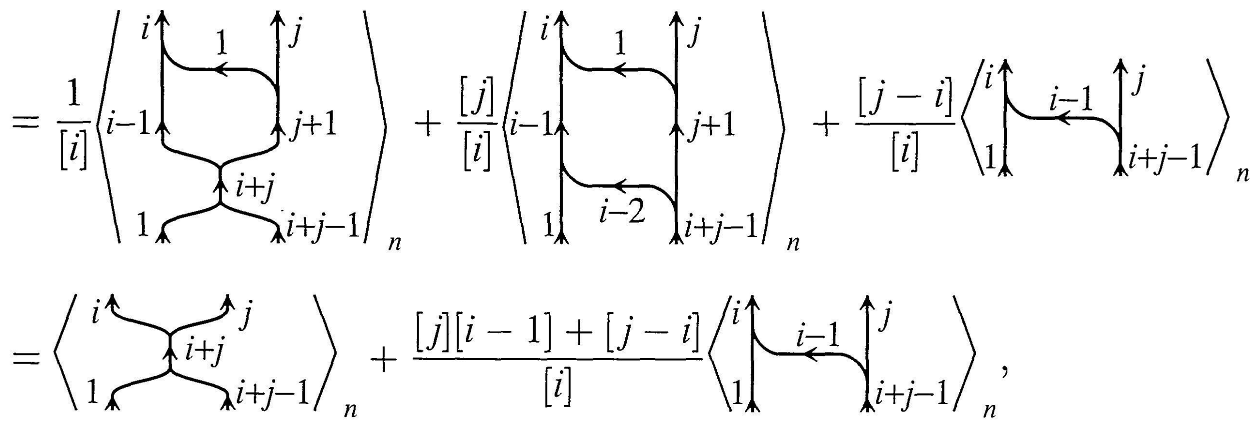
*Proof.* We first prove the case $k = 1$. We will show



by induction on $j$. Note that Lemma A.3 is the case $i = 1$.

The left hand side becomes

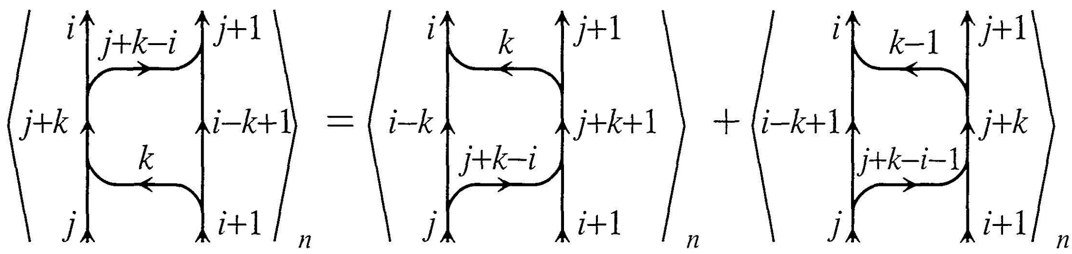





which is equal to the right hand side. Here we use Lemma A.6 in the first equality and the inductive hypothesis in the second equality.

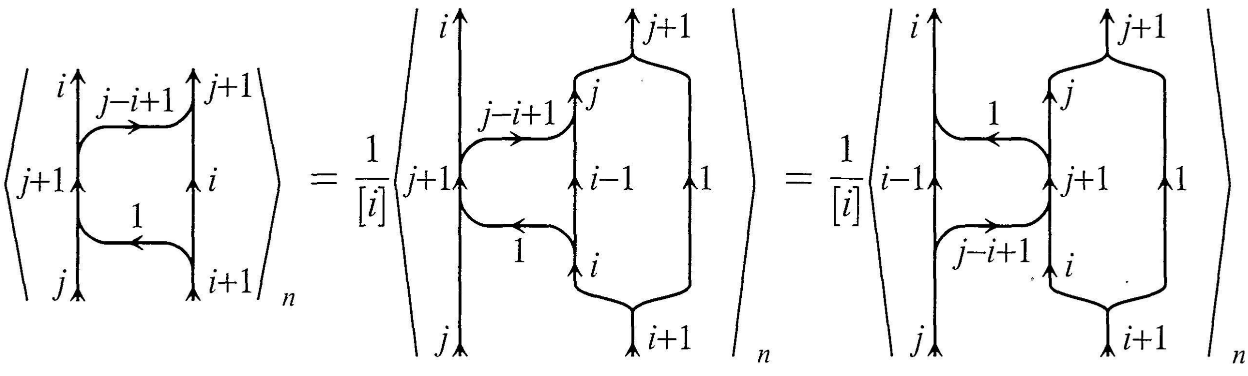
The proof for $k>1$ is similar and omitted. □

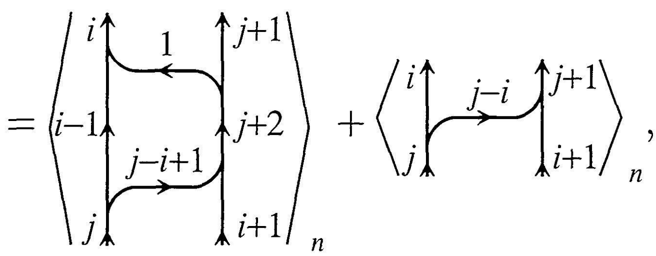
LEMMA A.8.



*for* $j \geq i \geq k \geq 1$.

*Proof.* For $k=1$ we have

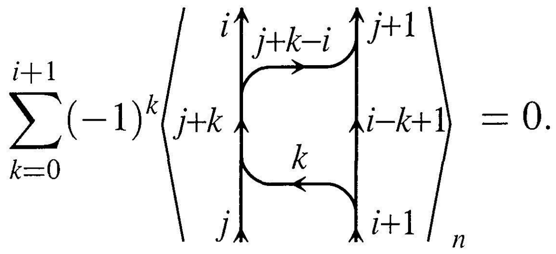




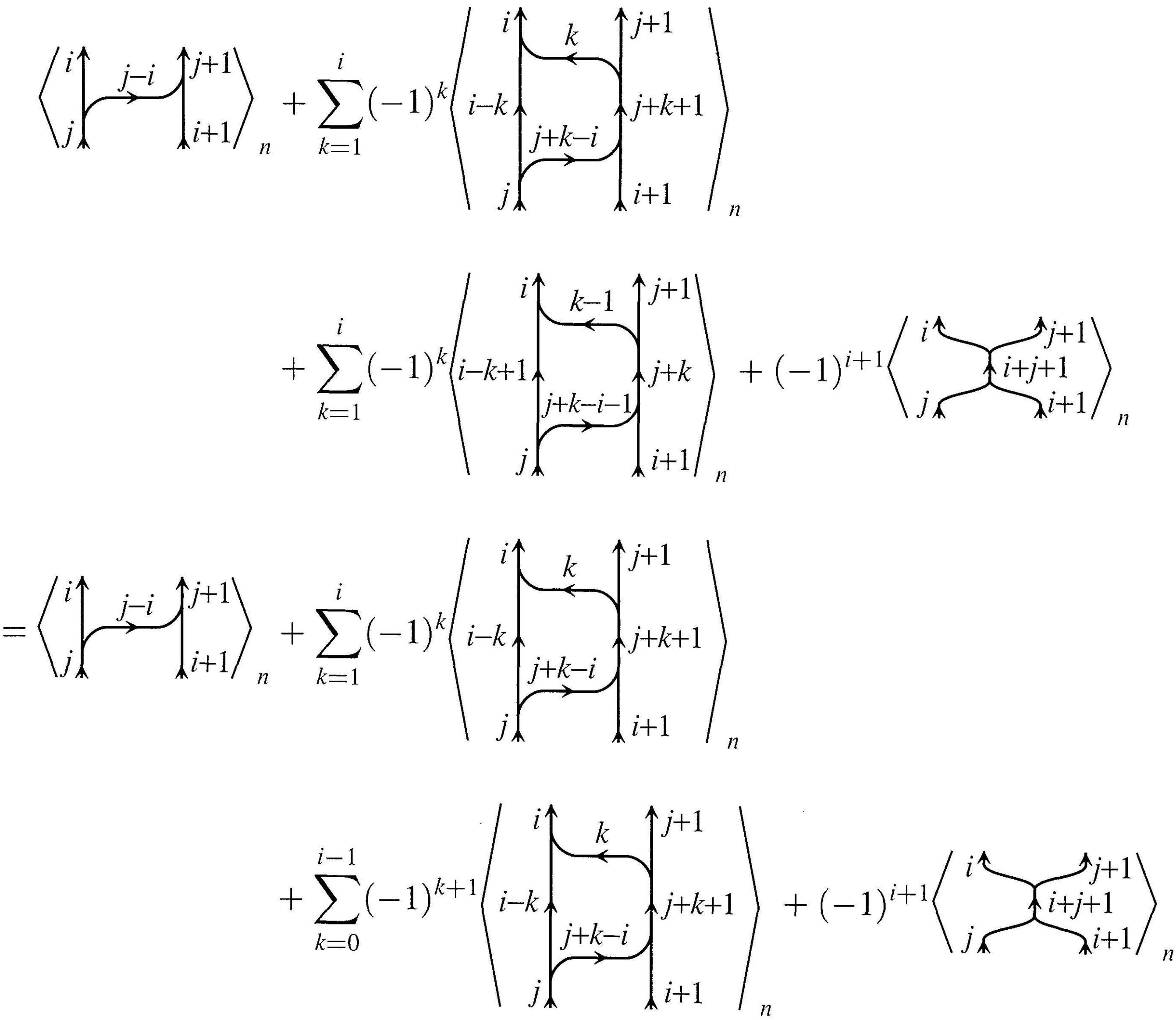
where we use Lemma A.7 in the last equality.

The proof for $k>1$ is similar and left to the reader. □

PROPOSITION A.9.



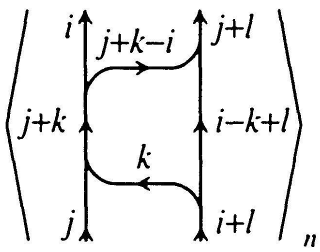

*Proof.* From Lemma A.8, we see that the left hand side equals



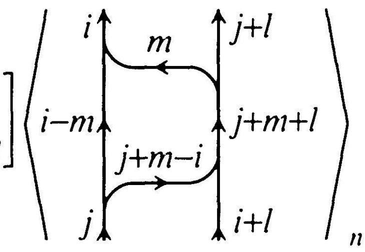

$= 0,$ and the proof is complete. □

More generally we have the following formula, which was suggested by J. Murakami. The proof is left to the reader.

PROPOSITION A.10.




$$= \sum_{m=0}^{i} \begin{bmatrix} l \\ k-m \end{bmatrix}$$




*for* $j \geq i \geq k \geq 1$*. Here* $\begin{bmatrix} x \\ y \end{bmatrix} = 0$ *if* $y < 0$ *or* $y > x$.