We now choose $p_1, q_1, r_1, p_2, q_2, r_2$ as follows:

$$\begin{cases} p_1 = a\alpha^5 - b\beta^5, & q_1 = 2a\alpha^4\beta, \quad r_1 = a\alpha^5 + b\beta^5, \\ p_2 = -c\alpha^5 + d\beta^5, & q_2 = -2c\alpha^4\beta, \quad r_2 = -c\alpha^5 - d\beta^5. \end{cases} \tag{4}$$

With these values of $p_1, q_1, r_1, p_2, q_2, r_2$ (and m_1, n_1, m_2, n_2 arbitrary), we find that the coefficients of uv^4 and u^2v^3 in equation (3) become zero. The coefficient of u^3v^2 in (3) will also vanish if

$$abm_1^3m_2^2(a^2\alpha^{10} - b^2\beta^{10}) - cdn_1^3n_2^2(c^2\alpha^{10} - d^2\beta^{10}) = 0. \tag{5}$$

We accordingly choose

$$\begin{cases} m_1 = n_2, \quad n_1 = m_2, \\ m_2 = ab(a^2\alpha^{10} - b^2\beta^{10}), \quad n_2 = cd(c^2\alpha^{10} - d^2\beta^{10}), \end{cases} \tag{6}$$

so that (5) is satisfied. Now, equation (3) has only the terms involving u^4v and u^5 and it is readily solved to give the following solution for u, v:

$$\begin{cases} u = -20a^2b\alpha^6m_1^4m_2(a^2\alpha^{10} - b^2\beta^{10}) \\ \qquad\qquad - 20c^2d\alpha^6n_1^4n_2(c^2\alpha^{10} - d^2\beta^{10}), \\ v = abm_1^5(11a^4\alpha^{20} - 10a^2b^2\alpha^{10}\beta^{10} - b^4\beta^{20}) \\ \qquad\qquad - cdn_1^5(11c^4\alpha^{20} - 10c^2d^2\alpha^{10}\beta^{10} - d^4\beta^{20}). \end{cases} \tag{7}$$

Thus, a parametric solution of equation (1) in terms of the parameters α and β is given by (2), where m_1, n_1, m_2, n_2 are defined by (6), $p_1, q_1, r_1, p_2, q_2, r_2$ by (4), and u, v by (7).

As a numerical example, when $a = 1$, $b = 2$, $c = 3$, $d = 6$, taking $\alpha = 1$, $\beta = 2$, we get the following solution of (1):

$$(X_1, X_2, X_3, X_4) = (1955587, 2963474, 121184667, 242404434),$$
$$(Y_1, Y_2, Y_3, Y_4) = (1022467, 2992634, 121219227, 242403354).$$

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