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## 2.2 A FUBINI COUNTEREXAMPLE

In this section, we give an example of (2). Let  $\mathcal{A}$  be as in (5) and define  $\mu|_{\mathcal{A}}$  by

$$(11) \quad \mu(A) := \begin{cases} 0 & \text{if } A \text{ is meager,} \\ 1 & \text{if } A \text{ is comeager.} \end{cases}$$

This is possible, since no set  $A \subset \mathbf{R}$  is simultaneously meager and comeager, for otherwise  $\emptyset = A \cap A^c$  would be comeager, in contradiction to Baire's theorem. It is easy to check that  $\mu$  is a probability measure on  $(\mathbf{R}, \mathcal{A})$ . Let again  $\nu := \lambda :=$  Lebesgue measure on  $\mathcal{B} := \mathcal{B}(\mathbf{R})$ , and choose  $A \in \mathcal{A}$  meager with  $\lambda(A^c) = 0$ . Then  $1_A(\cdot + y)$  is  $\mathcal{A}$ -measurable with

$$\int_{\mathbf{R}} 1_A(x + y) d\mu(x) = \mu(A - y) = 0 \quad (y \in \mathbf{R}).$$

On the other hand, we have

$$\int_{\mathbf{R}} 1_A(x + y) d\nu(y) = \lambda(A - x) = \infty \quad (x \in \mathbf{R}).$$

Hence (2) is obviously true in this case.

## 3. MEASURABILITY

Here is a positive result, having a certain measurability property of  $F$  from (1) among its conclusions. An application of this occurs in Mattner (1999).

3.1. THEOREM. *Let  $(\mathcal{X}, \mathcal{A}, \mu)$  and  $(\mathcal{Y}, \mathcal{B}, \nu)$  be  $\sigma$ -finite measure spaces, let  $f: \mathcal{X} \times \mathcal{Y} \rightarrow [0, \infty]$  be a function measurable with respect to the product  $\sigma$ -algebra  $\mathcal{A} \otimes \mathcal{B}$ , and put*

$$\mathcal{A}_0 := \sigma(\{f(\cdot, y) : y \in \mathcal{Y}\}),$$

$$\mathcal{B}_0 := \sigma(\{f(x, \cdot) : x \in \mathcal{X}\}),$$

$$\overline{\mathcal{A}_0} := \{A \in \mathcal{A} : \exists A_0 \in \mathcal{A}_0 \text{ with } A = A_0 \quad [\mu]\},$$

$$\overline{\mathcal{B}_0} := \{B \in \mathcal{B} : \exists B_0 \in \mathcal{B}_0 \text{ with } B = B_0 \quad [\nu]\},$$

$$\overline{\mathcal{A}_0 \otimes \mathcal{B}_0} := \{C \in \mathcal{A} \otimes \mathcal{B} : \exists C_0 \in \mathcal{A}_0 \otimes \mathcal{B}_0 \text{ with } C = C_0 \quad [\mu \otimes \nu]\}.$$

*Then  $f$  is  $\overline{\mathcal{A}_0 \otimes \mathcal{B}_0}$ -measurable,  $\int_{\mathcal{Y}} f(\cdot, y) d\nu(y)$  is  $\overline{\mathcal{A}_0}$ -measurable, and  $\int_{\mathcal{X}} f(x, \cdot) d\mu(x)$  is  $\overline{\mathcal{B}_0}$ -measurable.*