$P(2)$ into 8 triangles D_j so that a_j is a side of D_j, $j = 1, \ldots, 8$, compare Figure 7. Since M is hyperelliptic, D_j and D_{j+4} are isometric, $j = 1, \ldots, 4$. Denote by δ_i the angle of D_i in the vertex C, $i = 1, \ldots, 4$. The seven lengths determine the triangles D_i, $i = 1, 2, 3$, as well as two sides and the angle δ_4 of D_4 by the condition

$$\Delta := \sum_{j=1}^{4} \delta_j = \pi \,, \tag{6}$$

so they determine also D_4. This shows that the seven lengths determine $P(2)$. Multiply the seven lengths by a positive real t and assume that the seven new lengths also determine a canonical polygon $P_t(2)$. If $t > 1$, then δ_i, $i = 1, 2, 3$, are smaller in $P_t(2)$ than in $P(2)$ by Lemma 9, therefore, by (6), δ_4 is larger in $P_t(2)$ than in $P(2)$. It follows by Lemma 7 that the sum of the two other angles of D_4 is smaller in $P_t(2)$ than in $P(2)$. Since all angles in D_i, $i = 1, 2, 3$, are smaller in $P_t(2)$ than in $P(2)$ by Lemma 9, it follows that

$$\sum_{i=1}^{4} \alpha_i$$

is smaller in $P_t(2)$ than in $P(2)$. But this contradicts condition (II) of canonical polygons. An analogous contradiction follows if $t < 1$ proving thus that $t = 1$ and therefore the theorem. $\square$

REMARK. Theorem 16 is new. It is well known that $6g-6$ length functions can never parametrize T_g so that the situation of Theorem 16 is the best we can expect. It is not known whether $6g-5$ geodesic length functions, *taken as homogeneous parameters*, can parametrize T_g for $g \geq 3$.

REFERENCES

[1] BEARDON, A. F. *The Geometry of Discrete Groups.* Springer, 1983.

[2] BUSER, P. *Geometry and Spectra of Compact Riemann Surfaces.* Birkhäuser, Boston, 1992.

[3] COLDEWEY, H.-D. Kanonische Polygone endlich erzeugter Fuchsscher Gruppen. Dissertation, Bochum, 1971.

[4] FORD, L. *Automorphic Functions.* Chelsea, New York, 1929.

[5] IVERSEN, B. *Hyperbolic Geometry.* Cambridge University Press, 1992.

[6] JOST, J. *Compact Riemann Surfaces.* Springer, 1997.

[7] KATOK, S. *Fuchsian Groups.* The University of Chicago Press, 1992.

[8] LEHNER, J. Discontinuous groups and automorphic functions. *Math. Surveys, No. VIII*, AMS Providence, 1964.

[9] MASKIT, B. On Poincaré's theorem for fundamental polygons. *Advances in math. 7* (1971), 219–230.

[10] POINCARÉ, H. Théorie des groupes fuchsiens. *Acta math. 1* (1882), 1–62.

[11] DE RHAM, G. Sur les polygones générateurs de groupes fuchsiens. *L'Enseignement math. (2) 17* (1971), 49–61.

[12] SCHMUTZ, P. *Une paramétrisation de l'espace de Teichmüller de genre g donnée par $6g-5$ géodésiques explicites.* Sém. théorie spectrale et géométrie, Chambéry-Grenoble (1991–1992), 59–64.

[13] —— Die Parametrisierung des Teichmüllerraumes durch geodätische Längenfunktionen. *Comment. Math. Helv. 68* (1993), 278–288.

[14] SCHMUTZ SCHALLER, P. Geometric characterization of hyperelliptic Riemann surfaces. *Ann. Acad. Sci. Fenn. Math.* (to appear).

[15] SIEGEL, C. L. *Topics in Complex Function Theory.* Vol. II. Wiley Interscience, 1969.

[16] THURSTON, W. P. *Three-dimensional Geometry and Topology.* Vol. I. Princeton University Press, 1997.

[17] ZIESCHANG, H., E. VOGT and H.-D. COLDEWEY. *Flächen und ebene diskontinuierliche Gruppen.* Springer LNM 122, 1970.

[18] ZIESCHANG, H., E. VOGT and H.-D. COLDEWEY. *Surfaces and Planar Discontinuous Groups.* Springer LNM 835, 1980.

(Reçu le 6 janvier 1998; version révisée reçue le 26 octobre 1998)

Paul Schmutz Schaller

Institut de mathématiques
Université de Neuchâtel
Rue Emile-Argand 11
CH-2007 Neuchâtel
Switzerland
e-mail: Paul.Schmutz@maths.unine.ch

vide-leer-empty