The last comment in the proof above and our earlier remarks about actions on S^{2^t-1} also imply the following: If π acts freely and topologically on S^{2^t-1}, then π satisfies all pq-conditions. On S^{2s-1} with $2s \neq 2^t$ there are always free smooth actions by metacyclic groups $D_{p,q}$ violating the pq-conditions.

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REFERENCES

[1] BROWN, K. S. *Cohomology of Groups.* Springer, 1982.

[2] DAVIS, J. F. and R. J. MILGRAM. A survey of the spherical space form problem. *Mathematical Reports 2*, Part 2. Harwood Academic Publishers, 1985.

[3] GEIGES, H. and C. B. THOMAS. Contact structures on 7-manifolds. In: *Geometry, Topology, and Dynamics* (F. Lalonde, ed.). CRM Proc. Lecture Notes 15 (1998), 53–67.

[4] GEIGES, H. and C. B. THOMAS. Almost linear actions by finite groups on S^{2n-1}. In: *Proceedings of the Kirbyfest* (J. Hass and M. Scharlemann, eds.). Geometry & Topology Monographs 2 (1999), 135–156.

[5] KIRBY, R. Problems in low-dimensional topology. In: *Geometric Topology* (W. H. Kazez, ed.). AMS/IP Stud. Adv. Math. 2, Part 2. Amer. Math. Soc. (1997), 35–473.

[6] LAITINEN, E. and I. MADSEN. Topological classification of $SL_2(\mathbf{F}_p)$ space forms. In: *Algebraic Topology* (J. L. Dupont and I. H. Madsen, eds.). Lecture Notes in Math. 763 (1979), 1–27.

[7] MADSEN, I., C. B. THOMAS and C. T. C. WALL. The topological spherical space form problem III: Dimensional bounds and smoothings. *Pacific J. Math. 106* (1983), 135–143.

[8] MILNOR, J. Groups which act on S^n without fixed points. *Amer. J. Math. 79* (1957), 623–630.

[9] PETRIE, T. Free metacyclic group actions on homotopy spheres. *Ann. of Math. (2) 94* (1971), 108–124.

[10] RUBINSTEIN, J. H. An algorithm to recognize the 3-sphere. In: *Proceedings of the International Congress of Mathematicians (Zürich, 1994).* Birkhäuser, 1995.

[11] SUZUKI, M. On finite groups with cyclic Sylow subgroups for all odd primes. *Amer. J. Math. 77* (1955), 657–691.

[12] THOMAS, C. B. Almost linear actions by $SL_2(p)$ on S^{2n-1}. *Forum Math. 9* (1997), 751–760.

[13] THURSTON, W. P. Three-dimensional manifolds, Kleinian groups and hyperbolic geometry. *Bull. Amer. Math. Soc. (N.S.) 6* (1982), 357–381.

[14] WALL, C. T. C. *Surgery on Compact Manifolds* (2nd edition, ed. by A. A. Ranicki). Math. Surveys Monogr. 69, Amer. Math. Soc., 1999.

[15] WOLF, J. A. *Spaces of Constant Curvature.* McGraw-Hill, 1967.

[16] —— A contact structure for odd-dimensional spherical space forms. *Proc. Amer. Math. Soc. 19* (1968), 196.

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