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COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

(THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

DISCUSSION DOCUMENT FOR THE TWELFTH ICMI STUDY

THE FUTURE OF THE TEACHING AND LEARNING OF ALGEBRA

INTRODUCTION

This document introduces a new ICMI study entitled *The Future of the Teaching and Learning of Algebra*, to be held at the University of Melbourne (Australia) in December 2001. The intention is that the word 'Algebra' will be interpreted broadly to encompass the diversity of definitions around the world, extending beyond the standard curriculum in some countries. It will include, for example, algebra as a language for generalisation, abstraction and proof; algebra as a tool for problem solving through equation solving or graphing; for modelling with functions; and the way algebraic symbols and ideas are used in other parts of mathematics and other subjects. The principal interest of many participants is likely to be related to secondary school mathematics (ages 11–18) and algebra with real variables, but the study is also concerned with tertiary algebra (e.g. linear algebra and abstract algebra) and with algebra and its precursors for young children.

There are many reasons why it is timely to focus on the future of the teaching and learning of algebra. We are at a critical point when it is desirable to take stock of what has been achieved and to look forward to what should be done and what can be done. In many countries, increasing numbers of students are now receiving secondary education and this is causing every part of the mathematics curriculum to be scrutinised. For algebra, perhaps more than other parts of mathematics, concerns of equity and of relevance arise. As the language of higher mathematics, algebra is a gateway to future study and mathematically significant ideas, but it is often a wall that blocks the paths of many. Should algebra be made more accessible to more students by changing the amount or nature of what is taught? Many countries have already embarked on such changes, hoping to increase access and success. Alternatively, are these changes necessary: is algebra truly useful for the majority of people and, even if it is, will it be useful in the future?

An algebra curriculum that serves its students well in the coming century may look very different from an ideal curriculum from some years ago. The increased availability of computers and calculators will change what mathematics is useful as well as changing how mathematics is done. At the same time as challenging the content of what is taught, the technological revolution is also providing rich prospects for teaching and is offering students new paths to understanding. In the past two decades, a substantial body of research on the learning and teaching of many aspects of algebra has been established and there have been many experiments with adapting curricula and teaching methods. There is therefore a strong scientific basis upon which to build this study.

Outline of the program

The study has two aims: to make a synthesis of current thinking and lessons from the past which will help set directions for future work in the field, and to suggest guidelines for advancing the teaching and learning of algebra. Following the pattern of previous ICMI studies, this study will have two components: an invited study conference and a study volume to appear in the *ICMI Study Series*, which will share the findings with a broad international audience. A report will also be made at ICME-10 in 2004. The study conference program will therefore contain plenary and sub-plenary lectures, working groups and panels. At least two panels are planned. One will attempt to make explicit some perspectives on algebra, algebra activity, algebraic thinking or algebraic understanding. A second aims to highlight the significant differences in algebra education around the world and identify the main strands in the goals, content and teaching methods of this worldwide enterprise. A major part of the working time will be spent in working groups addressing different aspects of the study problem. Working groups are likely to be established to correspond with each of the sections listed below.

WHY ALGEBRA?

The technological future of a modern society depends in large part on the mathematical literacy of its citizens and this is reflected in the worldwide trend towards mass secondary education. For an individual, algebra is a gateway to much of higher education and therefore to many fields of employment. Educators also argue that algebra is part of cultural heritage and is needed for informed and critical citizenship. However, for many, algebra acts more like a wall than a gateway, presenting an obstacle that they find too difficult to cross. This section of the study is concerned with the significance of algebra for the broad population of secondary school students, recognising that regional and cultural differences may impact upon the answers in interesting ways. It addresses questions such as:

- Should algebra be taught to all ? There has been a call for algebra for all secondary students, but what aspects of algebra are of value to all ? What should comprise a minimal curriculum ? How do answers to these questions relate to regional or cultural differences ?
- What do we expect of an algebra-literate individual ? What are the values of algebra learning for the individual, especially in view of increasingly powerful computing capabilities ? Access to higher learning and employment are two values, but what are the more immediate values and how can they be achieved ?
- How can we reshape the algebra curriculum so that it has more immediate value to individuals ? Can we identify explicit examples in contexts meaningful to students in which algebraic ideas have clear, unambiguous value ? Are there undesirable consequences of such orientations to algebra ?
- How can we reshape the algebra curriculum so that specific difficult ideas are more easily accessed ?

APPROACHES TO ALGEBRA

Recent research has focused on a number of approaches for developing meaning for the objects and processes of algebra. These approaches include, but are not limited to, problem-solving approaches, functional approaches, generalisation approaches, language-based approaches, and so on. Problem-solving approaches tend to emphasise an analysis of problems in terms of equations and a view of letters as unknowns. Functional approaches support a different set of meanings for the objects of algebra; for instance, the use of expressions to represent relationships and an interpretation of letters in terms of quantities that vary. A somewhat different perspective is encouraged by generalisation approaches that stress expressions of generality to represent geometric patterns, numerical sequences, or the rules governing numerical relationships – such approaches often serving as a basis for exploring underlying numerical structure, predicting, justifying and proving. Some algebra curricula develop student algebraic thinking exclusively along the lines of one such approach throughout the several grades of secondary school; others attempt to combine facets of several approaches.

Synthesising the experience with and research on the use of various approaches in the teaching/learning of algebra leads to questions such as the following:

- What does each of these various teaching approaches mean ?
- What are the algebraic meanings supported by each ?
- What are the epistemological obstacles inherent in each ?
- Which important aspects of algebra are favoured/neglected in each approach ?
- What are the difficulties encountered by students in extending the meanings that are developed by each of these approaches to include the meanings inherent in other approaches ?

LANGUAGE ASPECTS OF ALGEBRA

This section considers theoretical and applied aspects of the languages and notations of algebra, in relation to teaching and learning. The evolution of algebra cannot be separated from the evolution of its language and notations. Historically the introduction of good notations has had enormous impact upon the development of algebra but a good notation for science may not be a good notation for learning. With new computer technology we are now seeing a flowering of new quasi-algebraic notations, which may offer, support or eventually enforce new notations. However, current theories of mathematics teaching and learning do not seem adequate to deal with learning about notation. It is therefore timely to focus on algebraic notations asking questions such as:

- How do theories of mathematics teaching and learning embrace the linguistic aspects of algebra and what can we propose to better take into account these aspects?
- Algebra is not a language but it has a language and the two cannot be dissociated. What does it mean to talk about algebra as a language and what are the implications of such a perspective?
- There is a wide range of theories of how mathematical concepts are learned and taught (in particular the constructivist theories) but learning a language is not just a matter of learning concepts. How do acknowledged theories of mathematics learning and teaching embrace the non-conceptual aspects of learning the language of algebra and what can we propose to better take these aspects into account?
- Would some changes of algebraic language contribute to the development of algebraic thinking, communication and understanding?
- Is it feasible and desirable to remove some of the ambiguities that are present in standard mathematical symbolism, for example in the use of the equals sign?
- Should some effort be made in the teaching of mathematics to explain and bridge differences in notation between algebra as it is taught in mathematics courses and algebra as it is used in other disciplines?
- What are the characteristics of good notation? What does mathematics education research have to say on this? Are some notational choices better for science but others better for learning?

TEACHING AND LEARNING WITH COMPUTER ALGEBRA SYSTEMS

The advent of affordable computer systems and calculators that can perform symbolic calculations may lead to far-reaching changes in mathematics curricula and in mathematics teaching. This section addresses questions that arise from the increasing accessibility of computer symbolic manipulation. Answers to these questions will draw upon established research on the teaching and learning of algebra as well as reporting on recent experimental work. They may suggest new directions for research, including:

- For which students and when is it appropriate to introduce the use of a computer algebra system? When do the advantages of using such a system outweigh the effort that must be put into learning to use it? Are there activities using such systems that can be profitably undertaken by younger students?

- What algebraic insights and 'symbol sense' does the user of a computer algebra system need and what insights does the use of the systems bring?
- A strength of computer algebra systems is that they support multiple representations of mathematical concepts. How can this be used well? Might it be over-used?
- What are the relationships and interactions between different approaches and philosophies of mathematics teaching with the use of computer algebra systems?
- Students using different computational tools solve problems and think about concepts differently. Teachers have more options for how they teach. What impact does this have on teaching and learning? Which types of system support which kinds of learning? Can these differences be characterised theoretically?
- What should an algebra curriculum look like in a country where computer algebra systems are freely available? What 'by hand' skills should be retained?

TECHNOLOGICAL ENVIRONMENTS

Recent research, curriculum development, and classroom practice have incorporated a number of technologies to help students develop meaning for various algebraic objects, ideas and processes. These include, but are not limited to, function graphers, spreadsheets, programming languages, one-line programming on calculators, and other specific computer software environments. [Here, we exclude computer algebra systems that are treated elsewhere.] In an attempt to characterise recent research and experience, this section will explore which aspects of specific computer/calculator environments are related to which kinds of algebra learning. This question will be explored in depth for specific examples of such technology, by addressing questions such as the following:

- For a given technological environment, what are the implicit assumptions regarding the underlying core aspects of algebra?
- Which important aspects of algebra are and are not touched upon by this environment?
- What kinds of algebra learning does this environment promote?
- What particular limitations are associated with the use of this environment and how can such limitations be dealt with?
- To what extent ought the goals of algebra education be affected by the availability of this technology?
- To which aspects of algebra learning does this particular technology make a distinctive, unique contribution?
- Are there documented long-term consequences of embedding this particular technology in an algebra curriculum, and if so, what are they?

Submissions for this section should include discussion of as many of the above sub-questions as possible, but with particular attention paid to the first two items above.

ALGEBRA WITH REAL DATA

Modelling the behaviour of real things with algebraic functions is fundamental to applications of mathematics. Using real data to teach about functions is therefore important in the curriculum, and can also be highly motivating for students. Moreover, new devices (such as data loggers) and new communications technologies (such as the internet) provide new opportunities for bringing real data into the classroom. Questions such as the following arise:

- What new opportunities for using real data have proved to be successful and how do they relate to research on students' learning of functions and other algebraic concepts ?
- What are the strengths and weaknesses of using real data and how are these best managed in the classroom and in the curriculum?
- A commitment to using real data may lead to significant changes in curriculum content and sequence, for example by giving prominence to the exponential function over the quadratic. What changes may be required and what are their consequences ?
- Interpreting real data can lead students and teachers to question why the world is as it is. What is the role of algebra education in the development of critical thinking about social issues such as economics, health and environment?

USING THE HISTORY OF ALGEBRA

The history of algebra has been used extensively to identify epistemological obstacles in the learning of algebra and to characterise ruptures in the development of algebraic notions. Drawing on the history (or histories) of algebra from around the world, this section aims to analyse significant contributions and the value of these previous uses and also to reflect on possible avenues for research based on new areas, including:

- the history of symbolism; that is, the history of ways of representing quantities and operations in calculations;
- the history of methods for solving problems;
- the history of methods for solving equations;
- the history of the interactions of algebra with other mathematical domains (such as geometry); and
- the development of the idea of algebraic structures.

EARLY ALGEBRA EDUCATION

This section encompasses two different readings of the title, being concerned with both the algebra education for young children – say age 6 and above – and also the initial steps in more formal algebra education, which happens in some countries when students are about 12 years old. An ongoing concern is the relationship between arithmetic and algebra. Previous research has documented ways in which students' limited arithmetical experience can constitute an obstacle to the learning of algebra, so that an earlier start might reduce the problem; approaches have been proposed to achieve that. On the other hand, a much favoured approach to initial algebra education is based on the view of school algebra as generalised arithmetic, in which case an earlier start may not be appropriate. The general point here is that different views on the relationship between arithmetic and algebra will probably result in different views on algebra education, and this most important fact is a central concern in this section. The interest in algebra education for students at an early age is recent, and so there are as yet only a few studies in this area. It is important that answers to the following questions be thoroughly research-based:

- How early is «early algebra» and what are the advantages and disadvantages of an early start? How do the answers to these questions link to views on cognitive development and on learning, and on cultural and educational traditions?
- What aspects of algebra and algebraic thinking should be part of an early algebra education? Since the symbolic aspect of algebra is so essential, its early introduction may be beneficial, but is an awareness of algebra as a method to solve problems (for example) more important?
- What are the consequences of an early start to algebra for teachers and teacher education?

TERTIARY ALGEBRA

Problems exist in the teaching and learning of tertiary algebra courses such as abstract algebra, linear algebra, and number theory. Some are similar to the problems of secondary algebra: students' difficulties with abstraction, concerns of relevance, what to do with computing technology, etc. Other problems such as proof-making or seeing the objects of calculus as algebraic objects seem particular to the tertiary level. The questions below are concerned with these issues of learning and teaching and also with the specific question of education for prospective teachers.

- What are the contributions of tertiary algebra courses to the education of prospective secondary mathematics teachers? How do secondary teachers perceive the value of their tertiary algebra courses to their teaching experience?
- Secondary algebra has been well researched, and specific obstacles have been found in making the transition from arithmetic thinking to algebraic thinking. Do tertiary level students similarly experience obstacles in making the transition from secondary-level algebraic thinking to that required for the tertiary level?
- Why are certain types of definitions difficult for students? For example, why are definitions given in terms of properties to be satisfied (for example, subspaces

and group automorphisms) so difficult for students? How can this problem be addressed?

- There are specific questions about specific aspects of specific courses in algebra; for example, why do students who seem competent in $\mathbf{R}^n$ have difficulty with more concrete questions in $\mathbf{R}^2$ and $\mathbf{R}^3$? How can such questions be resolved?
- How does symbolic logic (through statements, connectives, quantifiers, qualified statements, and arguments) affect students' proof-making and their view of the value of proof-making?
- Secondary school algebra seems to lead more directly to applied mathematical modelling at the tertiary level, rather than to abstract algebra. What is going on here?
- Should secondary students learn more about algebraic structure?

HOW TO PARTICIPATE

The study conference will be held at the University of Melbourne from December 10 to December 14, 2001. As is the normal practice for ICMI studies, participation in the study conference is by invitation, given on the basis of papers submitted. A submitted paper may address issues from a number of sections above but it should identify one section as the primary focus. The pre-proceedings will contain the submissions of all participants and will form the basis for the scientific work of the study conference. The study volume, published after the conference, will contain selected revised contributions and reports. Submissions should pay particular attention to implications for the future of the teaching and learning of algebra. The work may report the results of individual studies (completed or in progress), or offer well-argued opinions. Survey and overview articles are especially welcome.

Submissions are invited from all interested who will be able to make a sound contribution to a scientific meeting. New researchers in the field are especially encouraged to submit, as are those with significant responsibility for curriculum development and implementation. The study conference is a fine opportunity for international exchange, so participants from countries under-represented in mathematics education research meetings are very welcome to submit. We hope that interaction in this study of mathematics teachers from the early years to tertiary levels, mathematics educators and mathematicians will produce new insights and guidelines for future work.

Submissions should be a paper 5 to 8 pages in length and should reach the Program Chair at the address below by January 31, 2001. Camera ready copy for the pre-proceedings is required. All submissions must be in English, the language of the study conference. Further technical details about the format of submissions will be available on the study website (see below), which will be progressively updated with all study and travel information. The combined fee for registration and college accommodation is expected to be less than US$ 500.

The members of the International Program Committee are: Program Chair Kaye STACEY (Australia), Dave CARLSON (USA), Jean-Phillipe DROUHARD (France), Desmond FEARNLEY-SANDER (Australia), Toshiakira FUJII (Japan), Carolyn KIERAN (Canada), Barry KISSANE (Australia), Romulo LINS (Brazil), Teresa ROJANO (Mexico), Luis PUIG (Spain), Rosamund SUTHERLAND (UK), Bernard HODGSON (ex-officio, ICMI). Helen CHICK (Australia) is the conference secretary.

FOR FURTHER INFORMATION

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