4.2 CONCLUDING REMARKS

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **47 (2001)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **23.07.2024**

4.2 Concluding remarks

It would be interesting to provide discrete analogues of other "4-vertex type" theorems known in the smooth case, and to find their specifically discrete proofs. We give two examples.

The following statement is a discrete version of the celebrated Möbius theorem (in dimension 2, "flattening" means "inflection") – see [9]:

An embedded non-contractible closed polygon in $\mathbf{RP}^2$ *has at least* 3 *flattenings.*

The notion of flattening for a polygonal line extends, in an obvious way, from $\mathbf{RP}^d$ to the sphere S^d. One has the following statement:

An embedded closed polygon in S^2 *bisecting the area has at least* 4 *flattenings.*

In the smooth case this was proved by B. Segre [14] and by V. Arnold (see [1, 2]).

We are confident that these statements hold true and can be proved in a similar way as in the smooth case. However, a detailed discussion would go beyond the limits of this article.

In conclusion, let us formulate a conjecture. For $k \geq d+2$ the following statement is stronger than Theorem 3.11.

Conjecture 4.2. *A strictly convex polygon in* $\mathbf{RP}^d$ *that intersects a hyperplane with multiplicity* k *has at least* k *flattenings.*

In the smooth case this is precisely Barner's result in full generality [3]. Conjecture 4.2 would imply strengthenings of Theorems 2.2, 2.6 and 2.10 – see [15] for the smooth case. For instance, the following result would hold.

Let X *and* Y *be two* n*-tuples of points in* $\mathbf{RP}^1$ *(see Section 2.3). If the closed broken line* $\big((x_1,y_1),(x_2,y_2),\dots,(x_n,y_n)\big)$ *in* $\mathbf{RP}^1 \times \mathbf{RP}^1$ *intersects the graph of a projective transformation with multiplicity* k*, then there exist at least* k *extremal triples of indices.*

ACKNOWLEDGMENTS. This work was supported by the Volkswagen-Stiftung (RiP-program at Oberwolfach). We are grateful to the Mathematisches Forschungsinstitut at Oberwolfach for the creative atmosphere. The second author is also grateful to the Max-Planck Institut in Bonn for its hospitality. The second author was supported by an NSF grant.

REFERENCES

[1] ARNOLD, V. *Topological Invariants of Plane Curves and Caustics.* University Lecture Series 5, AMS 1994.

[2] —— Topological problems in the theory of wave propagation. *Russian Math. Surveys 51:1* (1996), 1–47.

[3] BARNER, M. Über die Mindestanzahl stationärer Schmiegebenen bei geschlossenen streng-konvexen Raumkurven. *Abh. Math. Sem. Univ. Hamburg 20* (1956), 196–215.

[4] BLASCHKE, W. *Vorlesungen über Differentialgeometrie*, Vol. 2. Springer-Verlag, 1923.

[5] CONNELY, R. Rigidity. In *Handbook of Convex Geometry*, 223–272. North-Holland, 1993.

[6] DUVAL, C. and V. OVSIENKO. Schwarzian derivative and Lorentzian world lines. *Funct. Anal. Appl. 34* (2000), 69–72.

[7] GHYS, E. *Cercles osculateurs et géométrie lorentzienne.* Talk at the Journée Inaugurale du CMI, 1995, Marseille.

[8] GUIEU, L., E. MOURRE and V. OVSIENKO. Theorem on six vertices of a plane curve via Sturm theory. *The Arnold-Gelfand Math. Seminars.* Birkhäuser, 1997, 257–266.

[9] MÖBIUS, A. F. Über die Grundformen der Linien der dritten Ordnung. *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften, Math.-Phys. Klasse I* (1852), 1–82.

[10] MUKHOPADHYAYA, S. New methods in the geometry of a plane arc. *Bull. Calcutta Math. Soc. 1* (1909), 31–37.

[11] OVSIENKO, V. and S. TABACHNIKOV. Sturm theory, Ghys theorem on zeroes of the Schwarzian derivative and flattening of Legendrian curves. *Selecta Math. (N. S.) 2* (1996), 297–307.

[12] SEDYKH, V. A theorem on four support vertices of a polygonal line. *Funct. Anal. Appl. 30* (1996), 216–218.

[13] —— Discrete versions of the four-vertex theorem. *Amer. Math. Soc. Transl. Ser. 2, 180* (1997), 197–207.

[14] SEGRE, B. Alcune proprietà differenziali in grande delle curve chiuse sghembe. *Rend. Mat. (6) 1* (1968), 237–297.

[15] TABACHNIKOV, S. On zeroes of the Schwarzian derivative. *Amer. Math. Soc. Transl. Ser. 2, 180* (1997), 229–239.

[16] —— A four-vertex theorem for polygons. *Amer. Math. Monthly 107* (2000), 830–833.