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NEW EXAMPLES OF MAXIMAL SURFACES

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ABSTRACT. We describe all closed hyperbolic triangle surfaces of a particularly simple type which are maximal, i.e. for which the length of the systole is a local maximum in Teichmüller space. We show that this class of triangle surfaces contains exactly three maximal surfaces. One of these surfaces is the well known Klein surface, the other two examples are new.

1. INTRODUCTION

A *Riemann surface of finite type* is a closed Riemann surface from which a finite number $m \geq 0$ of points, the so-called *punctures*, have been deleted. Closed Riemann surfaces (with no punctures) are topologically determined by their genus. In this note we only consider surfaces of genus $g \geq 2$ with $m \geq 0$ punctures. Such a surface admits a family of complete hyperbolic metrics of finite volume. Each of these metrics corresponds to precisely one complex structure of finite type.

The easiest way to describe all such hyperbolic metrics is to look at the *Teichmüller space* $\mathcal{T}_{g,m}$ of *marked* hyperbolic metrics of finite volume on a surface S_0 of genus g with m punctures. This Teichmüller space is the set of all pairs (f,h) where h is a hyperbolic metric on a surface S and f is the homotopy class of a homeomorphism $F\colon S_0 \to S$ of S_0 onto S. The *mapping*

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