Zeitschrift: L'Enseignement Mathématique

Band: 48 (2002)

Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: MM-SPACES AND GROUP ACTIONS

Kapitel: 8. Reading suggestions

Autor: Pestov, Vladimir

DOI: https://doi.org/10.5169/seals-66074

EXERCISE 15. Prove that $\underline{H}_1\mathcal{L}\iota$ is a metric on the space of (isomorphism classes of) all Polish mm-spaces.

EXERCISE 16. Prove that a sequence of mm-spaces $X_n = (X_n, d_n, \mu_n)$ forms a Lévy family if and only if it converges to the trivial mm-space in the metric $\underline{H}_1\mathcal{L}\iota$:

$$X_n \xrightarrow{\underline{H}_1\mathcal{L}\iota} \{*\}.$$

If one now replaces the trivial space on the right hand side with an arbitrary mm-space [6]), one obtains the concept of *concentration to a non-trivial space.*

According to Gromov, this type of concentration commonly occurs in statistical physics. At the same time, there are very few known non-trivial examples of this kind in the context of transformation groups.

Here is just one problem in this direction, suggested by Gromov. Every probability measure ν on a group G determines a random walk on G. How can one associate to (G, ν) in a natural way a sequence of mm-spaces which would concentrate to the boundary [Fur] of the random walk?

8. READING SUGGESTIONS

The 2001 Borel seminar was based on Chapter $3\frac{1}{2}$ of the green book [Gr3], which contains a wealth of ideas and concepts and can be complemented by [Gr4]. The survey [M3] by Vitali Milman, to whom we owe the present status of the concentration of measure phenomenon, is highly relevant and rich in material, especially if read in conjunction with a recent account of the subject by the same author [M4]. The book [M-S] is, in a sense, indispensable and should always be within one's reach. Talagrand's fundamental paper [Ta1] has to be at least browsed by every learner of the subject, while the paper [Ta2] of the same author offers an independent introduction to the subject of concentration of measure. The newly-published book by Ledoux [Led], apparently the first ever monograph devoted exclusively to concentration, is highly readable and covers a wide range of topics. Don't miss the introductory survey by Schechtman [Sch]. The modern setting for concentration was designed in the important paper [Gr-M1] by Gromov and Milman, which had also introduced the subject of this lecture and from which many results (perhaps with slight modifications) have been taken.

[6]) Or, more generally, a uniform space — for instance, a non-metrizable compact space — with measure.