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Artikel: ON THE CLASSIFICATION OF CERTAIN PIECEWISE LINEAR AND DIFFERENTIABLE MANIFOLDS IN DIMENSION EIGHT AND AUTOMORPHISMS OF $\sharp_{i=1}^b (S^2 \times S^5)$

Kapitel: 3.3 Homotopy vs. isotopy

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iii) If X comes with an orientation, we may assume that D^m is orientation preservingly embedded and that all attaching maps are orientation reversing.

3.2 Consequences for E-manifolds of dimension eight with $w_2 = 0$

Let X be a piecewise linear (smooth) E-manifold of dimension eight with $w_2(X) = 0$. The first observation concerns the structure of W_2.

Lemma 3.5. *One has* $W_2 \cong \#_{i=1}^b (S^2 \times D^6)$.

Proof. The manifold W_2 is an $(8,1)$-handle body and as such homeomorphic to the boundary connected sum of b D^6-bundles over S^2 ([17], § 11, p. 115). As $\pi_1(\mathrm{SO}(4)) \cong \mathbf{Z}_2$ and we have requested $w_2(X) = 0$, the claim follows. □

The next consequence is

The manifold W_4 *is determined by a framed link of* $b_4(X)$ *three-dimensional spheres in* $\#_{i=1}^b (S^2 \times S^5)$.

We shall look into the classification of such links below. The third consequence is

Lemma 3.6. i) $\partial W_4 \cong \#_{i=1}^b (S^2 \times S^5)$.

ii) *The manifold* X *is of the form* $W_4 \cup_f \#_{i=1}^b (S^2 \times D^6)$ *where*

$$f \colon \#_{i=1}^b (S^2 \times S^5) \longrightarrow \partial W_4$$

is a piecewise linear (smooth) isomorphism, such that f_* *maps the canonical basis of* $H_2(\#_{i=1}^b (S^2 \times S^5), \mathbf{Z})$ *to the preferred basis of* $H_2(\partial W^4, \mathbf{Z})$.

Proof. i) This follows because $\partial W_4 \cong \partial \overline{W}_2$. ii) follows because $X = W_4 \cup \overline{W}_2$, and $\overline{W}_2 \cong \#_{i=1}^b (S^2 \times D^6)$, by Lemma 3.5. □

3.3 Homotopy vs. isotopy

By Theorem 3.3, the manifold is determined by the ambient isotopy classes of the attaching maps. However, the topological invariants of the manifold give us at best their homotopy classes. It is, therefore, important to have theorems granting that this is enough. In the setting of differentiable manifolds, we have

THEOREM 3.7 (Haefliger [6], [7]). *Let S be a closed differentiable manifold of dimension n and M an m-dimensional differentiable manifold without boundary. Let $f\colon S \longrightarrow M$ be a continuous map and $k \geq 0$, such that*

$$\pi_i(f)\colon \pi_i(S) \longrightarrow \pi_i(M)$$

is an isomorphism for $0 \leq i \leq k$ and surjective for $i = k+1$. Then the following is satisfied:

1. *If $m \geq 2n-k$ and $n > 2k+2$, then f is homotopic to a differentiable embedding.*
2. *If $m > 2n-k$ and $n \geq 2k+2$, then two differentiable embeddings of S into M which are homotopic are also ambient isotopic.*

In the setting of piecewise linear manifolds, similar results hold true. We refer to Haefliger's survey article [9]. For our purposes, the result stated below will be sufficient.

THEOREM 3.8. *Suppose S is a closed n-dimensional manifold, M an m-dimensional manifold without boundary, and $f\colon S \longrightarrow M$ a continuous map. Assume one has*

- *$m - n \geq 3$;*
- *S is $(2n-m+1)$-connected;*
- *M is $(2n-m+2)$-connected.*

Then:

(1) *f is homotopic to an embedding;*

(2) *two embeddings which are homotopic to f are ambient isotopic.*

Proof. The theorem of Irwin ([27], Thm. 7.12 and Ex. 7.14, [13], Thm. 8.1) yields (1) and that f_1 and f_2 as in (2) are concordant. But, since $m-n \geq 3$, concordance implies ambient isotopy ([13], Chap. IX). □

COROLLARY 3.9. *Let $S := S^3$ and M a simply connected differentiable or piecewise linear manifold of dimension 7 without boundary. Then $\pi_3(M)$ classifies differentiable and piecewise linear embeddings, respectively, of S^3 into M up to ambient isotopy.*

COROLLARY 3.10 (Zeeman's unknotting theorem [39]). *For every m, n with $m-n \geq 3$, any piecewise linear embedding of S^n into S^m is isotopic to the standard embedding.*