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<b>Autor:</b>	Schmitt, Alexander
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### 3.4 SOME 4-DIMENSIONAL CW-COMPLEXES

By Remark 3.4, a handle decomposition of  $X$  gives us a CW-complex which is homotopy equivalent to  $X$ . The following discussion will enable us to understand the 4-skeleton of that complex.

Let  $W := S^2 \vee \cdots \vee S^2$  be the  $b$ -fold wedge product of 2-spheres. Suppose  $X$  is the CW-complex obtained by attaching a 4-cell to  $W$  via the map  $g \in \pi_3(W)$ . The Hilton-Milnor theorem ([30], Thm. 7.9.4) asserts that

$$\pi_3(W) = \bigoplus_{i=1}^b \pi_3(S^2) \oplus \bigoplus_{1 \leq i < j \leq b} \pi_3(S^3).$$

Choosing the standard generators for  $\pi_3(S^2)$  and  $\pi_3(S^3)$ , we can describe  $g$  by a tuple  $(l_i, i = 1, \dots, b; l_{ij}, 1 \leq i < j \leq b)$  of integers. These integers determine the cohomology ring of  $X = W \cup_g D^4$  as follows:

**PROPOSITION 3.11.** *Let  $y \in H^4(X, \mathbf{Z})$  be the generator of  $H^4(X, \mathbf{Z})$  given by the attached 4-cell and  $x_1, \dots, x_b$  the canonical basis of  $H^2(X, \mathbf{Z}) = H^2(W, \mathbf{Z})$ . Then*

$$\begin{aligned} x_i \cup x_j &= l_{ij} \cdot y, \quad 1 \leq i < j \leq b, \\ x_i \cup x_i &= l_i \cdot y, \quad i = 1, \dots, b. \end{aligned}$$

This is proved like [22], (1.5), p. 103. We recall the proof in the following example.

**EXAMPLE 3.12.** We treat the case  $b = 2$ . Consider the embedding

$$\iota: S^2 \vee S^2 \hookrightarrow S^2 \times S^2 \hookrightarrow \mathbf{CP}^\infty \times \mathbf{CP}^\infty.$$

The standard basis for  $H^4(\mathbf{CP}^\infty \times \mathbf{CP}^\infty, \mathbf{Z}) \cong \mathbf{Z}^{\oplus 3}$  is given by the elements  $y_1, y_2, y_3$  obtained from attaching  $D^4$  via  $(1, 0; 0)$ ,  $(0, 0; 1)$ , and  $(0, 1; 0)$ , respectively. Let  $h: D^4 \longrightarrow D^4 \vee D^4 \vee D^4$  be the canonical map followed by

$$(\vartheta \cdot x \longmapsto \vartheta \cdot m_{l_1}(x)) \vee (\vartheta \cdot x \longmapsto \vartheta \cdot m_{l_{12}}(x)) \vee (\vartheta \cdot x \longmapsto \vartheta \cdot m_{l_2}(x)).$$

Here,  $m_k$  stands for a representative of  $[k \cdot \text{id}_{S^3}] \in \pi_3(S^3)$  and  $D^4 = \{ \vartheta \cdot x \mid x \in S^3, \vartheta \in [0, 1] \}$ . Now,  $h$  and  $\iota$  glue to a map  $f: X \longrightarrow \mathbf{CP}^\infty \times \mathbf{CP}^\infty$ , and

$$\begin{aligned} f^*: H^4(\mathbf{CP}^\infty \times \mathbf{CP}^\infty, \mathbf{Z}) &\longrightarrow H^4(X, \mathbf{Z}) \\ a_1 y_1 + a_2 y_2 + a_3 y_3 &\longmapsto (a_1 l_1 + a_2 l_{12} + a_3 l_2) y, \end{aligned}$$

so that the assertion follows from the naturality of the cup-product.