where, for each $i' \in \Lambda'$, $P(i') := \bigoplus_{n \in \mathbb{N}} P_{(n,i')}$. By Lemma 11, each $P(i')$ is free, so by (13), P is also free. This proves the first part of the theorem.

For the second part, let P be any *non countably generated* projective R-module. By Kaplansky's theorem in [Ka$_3$], we can express P in the form $\bigoplus_{i \in \Lambda} P_i$ (for some indexing set Λ), where the P_i's are nonzero countably generated projective R-modules. Since P itself is not countably generated, Λ must be an infinite set. Thus, the first part of the theorem applies, showing that P is free. $\square$

It seems plausible that, under the assumptions on R in Theorem 12, any countably but not finitely generated projective R-module P is also free. This would follow from Lemma 11 if we can decompose P as in that Lemma. However, we are not able to prove the existence of such a decomposition.

We close by recalling that most results in this note required the small 0-divisor assumption on R. The study of projective modules over general Prüfer rings (without the small 0-divisor assumption) awaits further effort.

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