We can now obtain Corollary 1.4 stated in the Introduction.

COROLLARY 5.4. *Let $G = \langle x_1, \dots, x_k \mid r_1, \dots, r_m \rangle$ be a nonelementary word-hyperbolic group and let $H \leq G$ be a quasiconvex subgroup of infinite index. Let a_n be the number of freely reduced words in $A = \{x_1, \dots, x_k\}^{\pm 1}$ of length n that represent elements of H. Let b_n be the number of all words in A of length n that represent elements of H. Then*

$$\limsup_{n \to \infty} \sqrt[n]{a_n} < 2k - 1$$

and

$$\limsup_{n \to \infty} \sqrt[n]{b_n} < 2k .$$

Proof. Note that $k \geq 2$ since G is nonelementary. Put $A = \{x_1, \dots, x_k\}$ and $Y = \Gamma(G, H, A)$. We choose $x_0 := H1 \in VY$ as the base-vertex of Y. Note that Y is $2k$-regular by construction. Also, for any vertex x of Y and any word w in $A \cup A^{-1}$ there is a unique path in Y with label w and origin x. The definition of Schreier coset graphs also implies that a word w represents an element of H if and only if the unique path in Y with origin x_0 and label w terminates at x_0. Therefore $a_n(Y)$ equals the number of freely reduced words in the alphabet $A = \{x_1, \dots, x_k\}^{\pm 1}$ of length n that represent elements of H. Similarly, $b_n(Y)$ equals the number of all words in A of length n representing elements of H. By Theorem 1.2, Y is nonamenable. Hence by Theorem 2.5, $\alpha(Y) < 2k - 1$ and $\beta(Y) < 2k$, as required.

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