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HYPERBOLICITY OF MAPPING-TORUS GROUPS AND SPACES

by François GAUTERO

ABSTRACT. This paper deals with the geometry of metric ‘two-dimensional’ spaces, equipped with semi-flows admitting transverse foliations by forests. Our main theorem relates the Gromov-hyperbolicity of such spaces, for instance mapping-telescopes of \mathbf{R} -trees, with the dynamical behaviour of the semi-flow. As a corollary, we give a new proof of the following theorem [3]: *Let α be a hyperbolic injective endomorphism of the rank n free group F_n . If the image of α is a malnormal subgroup of F_n , then $G_\alpha = F_n \rtimes_\alpha \mathbf{Z}$ is a hyperbolic group.*

INTRODUCTION

The subject of 3-dimensional topology changed completely in the seventies with Thurston’s geometric methods. His geometrization conjecture involves eight classes of manifolds, among which the hyperbolic manifolds play the most important role. In this context, a hyperbolic manifold is a compact manifold which admits (or whose interior admits in the case of non-empty boundary) a metric of constant curvature -1 . According to another conjecture of Thurston, any closed hyperbolic 3-manifold should have a finite cover which is a mapping-torus. This gives a particular interest to these mapping-tori manifolds. Recall that a mapping-torus is a manifold which fibers over the circle. Namely this is a 3-manifold constructed from a homeomorphism h of a compact surface Σ as

$$M = (\Sigma \times [0, 1]) / ((x, 1) \sim (h(x), 0)).$$

For these manifolds, the hyperbolization conjecture has been proved, see for instance [25]: the manifold M constructed from Σ and h as above is hyperbolic if and only if Σ has negative Euler characteristic and h is a pseudo-Anosov homeomorphism (see [12]).