

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 49 (2003)  
**Heft:** 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** HYPERBOLICITY OF MAPPING-TORUS GROUPS AND SPACES  
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**Kapitel:** 5.2 Straight telescopic paths  
**DOI:** <https://doi.org/10.5169/seals-66690>

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LEMMA 5.4. *Let  $p$  be a horizontal geodesic which admits a decomposition in  $r$  subpaths  $p_i$  such that for some constant  $L \geq 0$ , for any  $i = 1, \dots, r$ , either  $|[p_i]_{r+nt_0}|_{r+nt_0} \leq |p_i|_r$  or  $L \geq |[p_i]_{r+nt_0}|_{r+nt_0} > |p_i|_r$ . Then there exists a constant  $C_{5.4}(n, r, L)$ , which is increasing in each variable, such that if  $p$  is dilated in the future after  $nt_0$ , then  $|p|_r \leq C_{5.4}(n, r, L)$ .*

*Proof.* We set  $n = 1$  in order to simplify the notation; the general case is treated in the same way. Up to permuting the indices,  $|[p_i]_{r+t_0}|_{r+t_0} > |p_i|_r$  for  $i = 1, \dots, j$ . Since  $p$  is dilated in the future after  $t_0$ ,

$$jL + \sum_{i=j+1}^r |p_i|_r \geq \lambda^{t_0} \sum_{i=1}^r |p_i|_r.$$

Therefore  $|p|_r \leq \frac{jL}{\lambda^{t_0} - 1}$ .  $\square$

## 5.2 STRAIGHT TELESCOPIC PATHS

DEFINITION 5.5. A *straight* telescopic path is a telescopic path  $S$  such that if  $x, y$  are any two points in  $S$  with  $x \in O^+(y) \cup O^-(y)$  then the subpath of  $S$  between  $x$  and  $y$  is equal to the orbit-segment of the semi-flow between  $x$  and  $y$ .

If  $S$  is a path containing a point  $x$ , let  $S_{x,t} \subset S$  be the maximal subpath of  $S$  containing  $x$ , whose pulled-tight projection  $[S_{x,t}]_{f(x)+t}$  on  $f^{-1}(f(x)+t)$  is well defined. The point  $\sigma_t(x)$  does not necessarily belong to  $[S_{x,t}]_{f(x)+t}$ . However there exists a unique point in  $[S_{x,t}]_{f(x)+t}$  which minimizes the horizontal distance between  $\sigma_t(x)$  and  $[S_{x,t}]_{f(x)+t}$ . This point is denoted by  $\bar{x}_t$ . Lemma 5.6 below gives an upper bound, depending on  $t$ , for the telescopic distance between  $x$  and  $\bar{x}_t$ .

LEMMA 5.6. *Let  $S$  be any straight telescopic path. If  $t$  is any non negative real number, there exists a constant  $C_{5.6}(t) \geq t$ , which increases with  $t$ , such that any point  $x \in S$  is at telescopic distance smaller than  $C_{5.6}(t)$  from the point  $\bar{x}_t$  (see above).*

*Proof.* If  $\sigma_t(x) \in [S_{x,t}]_{f(x)+t}$ , we set  $C_{5.6}(t) = t$ . Since  $S$  is straight, if  $\sigma_t(x) \notin [S_{x,t}]_{f(x)+t}$ ,  $x$  belongs to a cancellation  $c$  whose endpoints lie in the past orbits of  $\bar{x}_t$ . The bounded-cancellation property gives an upper bound on the horizontal length of  $c$ . This leads to the conclusion.  $\square$