Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 49 (2003)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP
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Kapitel: 5.1 Notation
DOI: https://doi.org/10.5169/seals-66691

5. The basic gerbe over a compact simple Lie group

In this section we explain our construction of the basic gerbe over a compact, simple, simply connected Lie group.

5.1 Notation

Let G be a compact, simple, simply connected Lie group, with Lie algebra $\mathfrak{g}$. For any action of $G \times M \to M$, $(g,m) \mapsto g.m$ on a manifold M, we will denote by G_m the stabilizer group of a point $m \in M$. If $M = G$ or $M = \mathfrak{g}$, we will always consider the adjoint action of G unless specified otherwise. For instance, G_g for denotes the centralizer of an element $g \in G$.

Choose a maximal torus T of G, with Lie algebra $\mathfrak{t}$. Let $\Lambda = \ker(\exp|_{\mathfrak{t}})$ be the integral lattice and $\Lambda^* \subset \mathfrak{t}^*$ its dual, the (real) weight lattice. Equivalently, Λ is characterized as the lattice generated by the coroots $\check{\alpha}$ for the (real) roots α. Recall that the *basic inner product* $\cdot$ on $\mathfrak{g}$ is the unique invariant inner product such that $\check{\alpha} \cdot \check{\alpha} = 2$ for all long roots α. Throughout this paper, we will use the basic inner product to identify $\mathfrak{g}^* \cong \mathfrak{g}$. Choose a collection of simple roots $\alpha_1, \ldots, \alpha_d \in \Lambda^*$ and let $\mathfrak{t}_+ = \{\xi \mid \alpha_j \cdot \xi \geq 0,\ j = 1, \ldots, d\}$ be the corresponding positive Weyl chamber. The fundamental alcove $\mathfrak{A}$ is the subset cut out from $\mathfrak{t}_+$ by the additional inequality $\alpha_0 \cdot \xi \geq -1$ where α_0 is the lowest root.

The fundamental alcove parametrizes conjugacy classes in G, in the sense that each conjugacy class contains a unique point $\exp \xi$ with $\xi \in \mathfrak{A}$. The quotient map will be denoted $q\colon G \to \mathfrak{A}$. Let $\mu_0, \ldots, \mu_d$ be the vertices of $\mathfrak{A}$, with $\mu_0 = 0$. For any $I \subseteq \{0, \ldots, d\}$, all group elements $\exp \xi$ with ξ in the open face spanned by μ_j with $j \in I$ have the same centralizer, denoted G_I. In particular, G_j will denote the centralizer of $\exp \mu_j$.

For each j let $\mathfrak{A}_j \subset \mathfrak{A}$ be the open star at μ_j, i.e. the union of all open faces containing μ_j in their closure. Put differently, $\mathfrak{A}_j$ is the complement of the closed face opposite to the vertex μ_j. We will work with the open cover of G given by the pre-images, $V_j = q^{-1}(\mathfrak{A}_j)$. More generally let $\mathfrak{A}_I = \bigcap_{j \in I} \mathfrak{A}_j$, and $V_I := q^{-1}(\mathfrak{A}_I)$. The flow-out $S_I = G_I . \exp(\mathfrak{A}_I)$ of $\exp(\mathfrak{A}_I) \subset T$ under the action of G_I is an open subset of G_I, and is a slice for the conjugation action of G. That is,

$$G \times_{G_I} S_I = V_I\,.$$

We let $\pi_I\colon V_I \to G/G_I$ denote the projection to the base.