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## 5. THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP

In this section we explain our construction of the basic gerbe over a compact, simple, simply connected Lie group.

### 5.1 NOTATION

Let  $G$  be a compact, simple, simply connected Lie group, with Lie algebra  $\mathfrak{g}$ . For any action of  $G \times M \rightarrow M$ ,  $(g, m) \mapsto g.m$  on a manifold  $M$ , we will denote by  $G_m$  the stabilizer group of a point  $m \in M$ . If  $M = G$  or  $M = \mathfrak{g}$ , we will always consider the adjoint action of  $G$  unless specified otherwise. For instance,  $G_g$  for denotes the centralizer of an element  $g \in G$ .

Choose a maximal torus  $T$  of  $G$ , with Lie algebra  $\mathfrak{t}$ . Let  $\Lambda = \ker(\exp|_{\mathfrak{t}})$  be the integral lattice and  $\Lambda^* \subset \mathfrak{t}^*$  its dual, the (real) weight lattice. Equivalently,  $\Lambda$  is characterized as the lattice generated by the coroots  $\check{\alpha}$  for the (real) roots  $\alpha$ . Recall that the *basic inner product*  $\cdot$  on  $\mathfrak{g}$  is the unique invariant inner product such that  $\check{\alpha} \cdot \check{\alpha} = 2$  for all long roots  $\alpha$ . Throughout this paper, we will use the basic inner product to identify  $\mathfrak{g}^* \cong \mathfrak{g}$ . Choose a collection of simple roots  $\alpha_1, \dots, \alpha_d \in \Lambda^*$  and let  $\mathfrak{t}_+ = \{\xi \mid \alpha_j \cdot \xi \geq 0, j = 1, \dots, d\}$  be the corresponding positive Weyl chamber. The fundamental alcove  $\mathfrak{A}$  is the subset cut out from  $\mathfrak{t}_+$  by the additional inequality  $\alpha_0 \cdot \xi \geq -1$  where  $\alpha_0$  is the lowest root.

The fundamental alcove parametrizes conjugacy classes in  $G$ , in the sense that each conjugacy class contains a unique point  $\exp \xi$  with  $\xi \in \mathfrak{A}$ . The quotient map will be denoted  $q: G \rightarrow \mathfrak{A}$ . Let  $\mu_0, \dots, \mu_d$  be the vertices of  $\mathfrak{A}$ , with  $\mu_0 = 0$ . For any  $I \subseteq \{0, \dots, d\}$ , all group elements  $\exp \xi$  with  $\xi$  in the open face spanned by  $\mu_j$  with  $j \in I$  have the same centralizer, denoted  $G_I$ . In particular,  $G_j$  will denote the centralizer of  $\exp \mu_j$ .

For each  $j$  let  $\mathfrak{A}_j \subset \mathfrak{A}$  be the open star at  $\mu_j$ , i.e. the union of all open faces containing  $\mu_j$  in their closure. Put differently,  $\mathfrak{A}_j$  is the complement of the closed face opposite to the vertex  $\mu_j$ . We will work with the open cover of  $G$  given by the pre-images,  $V_j = q^{-1}(\mathfrak{A}_j)$ . More generally let  $\mathfrak{A}_I = \cap_{j \in I} \mathfrak{A}_j$ , and  $V_I := q^{-1}(\mathfrak{A}_I)$ . The flow-out  $S_I = G_I \cdot \exp(\mathfrak{A}_I)$  of  $\exp(\mathfrak{A}_I) \subset T$  under the action of  $G_I$  is an open subset of  $G_I$ , and is a slice for the conjugation action of  $G$ . That is,

$$G \times_{G_I} S_I = V_I.$$

We let  $\pi_I: V_I \rightarrow G/G_I$  denote the projection to the base.