

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 49 (2003)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP
Autor: Meinrenken, Eckhard
Kapitel: 5.1 Notation
DOI: <https://doi.org/10.5169/seals-66691>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 06.02.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

5. THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP

In this section we explain our construction of the basic gerbe over a compact, simple, simply connected Lie group.

5.1 NOTATION

Let G be a compact, simple, simply connected Lie group, with Lie algebra \mathfrak{g} . For any action of $G \times M \rightarrow M$, $(g, m) \mapsto g.m$ on a manifold M , we will denote by G_m the stabilizer group of a point $m \in M$. If $M = G$ or $M = \mathfrak{g}$, we will always consider the adjoint action of G unless specified otherwise. For instance, G_g for denotes the centralizer of an element $g \in G$.

Choose a maximal torus T of G , with Lie algebra \mathfrak{t} . Let $\Lambda = \ker(\exp|_{\mathfrak{t}})$ be the integral lattice and $\Lambda^* \subset \mathfrak{t}^*$ its dual, the (real) weight lattice. Equivalently, Λ is characterized as the lattice generated by the coroots $\check{\alpha}$ for the (real) roots α . Recall that the *basic inner product* \cdot on \mathfrak{g} is the unique invariant inner product such that $\check{\alpha} \cdot \check{\alpha} = 2$ for all long roots α . Throughout this paper, we will use the basic inner product to identify $\mathfrak{g}^* \cong \mathfrak{g}$. Choose a collection of simple roots $\alpha_1, \dots, \alpha_d \in \Lambda^*$ and let $\mathfrak{t}_+ = \{\xi \mid \alpha_j \cdot \xi \geq 0, j = 1, \dots, d\}$ be the corresponding positive Weyl chamber. The fundamental alcove \mathfrak{A} is the subset cut out from \mathfrak{t}_+ by the additional inequality $\alpha_0 \cdot \xi \geq -1$ where α_0 is the lowest root.

The fundamental alcove parametrizes conjugacy classes in G , in the sense that each conjugacy class contains a unique point $\exp \xi$ with $\xi \in \mathfrak{A}$. The quotient map will be denoted $q: G \rightarrow \mathfrak{A}$. Let μ_0, \dots, μ_d be the vertices of \mathfrak{A} , with $\mu_0 = 0$. For any $I \subseteq \{0, \dots, d\}$, all group elements $\exp \xi$ with ξ in the open face spanned by μ_j with $j \in I$ have the same centralizer, denoted G_I . In particular, G_j will denote the centralizer of $\exp \mu_j$.

For each j let $\mathfrak{A}_j \subset \mathfrak{A}$ be the open star at μ_j , i.e. the union of all open faces containing μ_j in their closure. Put differently, \mathfrak{A}_j is the complement of the closed face opposite to the vertex μ_j . We will work with the open cover of G given by the pre-images, $V_j = q^{-1}(\mathfrak{A}_j)$. More generally let $\mathfrak{A}_I = \bigcap_{j \in I} \mathfrak{A}_j$, and $V_I := q^{-1}(\mathfrak{A}_I)$. The flow-out $S_I = G_I \cdot \exp(\mathfrak{A}_I)$ of $\exp(\mathfrak{A}_I) \subset T$ under the action of G_I is an open subset of G_I , and is a slice for the conjugation action of G . That is,

$$G \times_{G_I} S_I = V_I.$$

We let $\pi_I: V_I \rightarrow G/G_I$ denote the projection to the base.