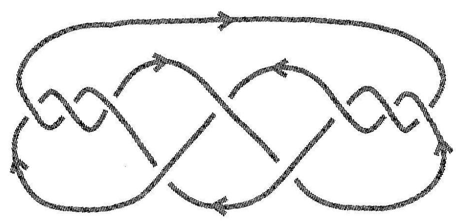
that the rational tangle has two components), the integer $u = 2K - q$ in the equation $q^2 = 1 + up$ cannot be zero. It follows then that the links of the type $N([2n])$, for $n \in \mathbf{Z}$, $n \neq 0$, with tangle fraction $2n/1$ are not invertible (recall the example in Figure 30). Note that, for $n = 0$ we have $T = [0]$ and $F(T) = 0/1$, and in this case Theorem 7 is confirmed, since $1^2 = 1 + u0$, for any u odd. See Figure 38 for another example of a strongly invertible link. In this case the link is $L = N([[3],[1],[1],[1],[3]])$ with $F(L) = 40/11$. Note that $11^2 = 1 + 3 \cdot 40$, fitting the conclusion of Theorem 7.



L = N([[3], [1], [1], [1], [3]])

FIGURE 38
An example of a strongly invertible link

ACKNOWLEDGEMENTS. A part of this work and the work in [15] was done at Göttingen Universität. Both authors acknowledge with gratitude the research facilities offered there. It also gives us pleasure to acknowledge a list of places and meetings where we worked on these matters. These are: Göttingen, Santa Fe, Chicago, Austin, Toulouse, Korea, Athens, Marne-la-Vallée, Siegen, Las Vegas, Marseille, Berkeley, Dresden, Potsdam NY.

REFERENCES

[1] BANKWITZ, C. und H. G. SCHUMANN. Über Viergeflechte. *Abh. Math. Sem. Univ. Hamburg 10* (1934), 263–284.

[2] BLEILER, S. and J. MORIAH. Heegaard splittings and branched coverings of B^3. *Math. Ann. 281* (1988), 531–543.

[3] BRODY, E. J. The topological classification of the lens spaces. *Ann. of Math. (2) 71* (1960), 163–184.

[4] BURDE, G. Verschlingungsinvarianten von Knoten und Verkettungen mit zwei Brücken. *Math. Z. 145* (1975), 235–242.

[5] BURDE, G. and H. ZIESCHANG. *Knots.* De Gruyter Studies in Mathematics *5*, 1985.

[6] CONWAY, J. H. An enumeration of knots and links and some of their algebraic properties. In: *Proceedings of the Conference on Computational Problems in Abstract Algebra (Oxford 1967)*, J. Leech ed., 329–358. Pergamon Press, 1970.

[7] COZZARELLI, N., F. DEAN, T. KOLLER, M. A. KRASNOW, S. J. SPENGLER and A. STASIAK. Determination of the absolute handedness of knots and catenanes of DNA. *Nature 304* (1983), 550–560.

[8] ERNST, C. and D. W. SUMNERS. The growth of the number of prime knots. *Math. Proc. Cambridge Philos. Soc. 102* (1987), 303–315.

[9] ERNST, C. and D. W. SUMNERS. A calculus for rational tangles: Applications to DNA recombination *Math. Proc. Cambridge Philos. Soc. 108* (1990), 489–515.

[10] ERNST, C. and D. W. SUMNERS. Solving tangle equations arising in a DNA recombination model. *Math. Proc. Cambridge Philos. Soc. 126* (1999), 23–36.

[11] FRAME, J. S. Continued fractions and matrices. Classroom notes, C. B. Allendoerfer ed. *Amer. Math. Monthly 56* (1949), 98–103.

[12] GOERITZ, L. Bemerkungen zur Knotentheorie. *Abh. Math. Sem. Univ. Hamburg 10* (1934), 201–210.

[13] GOLDMAN, J. R. and L. H. KAUFFMAN. Rational tangles. *Adv. in Appl. Math. 18* (1997), 300–332.

[14] GRGETA, E. Simony, Knots, and continued fractions. Senior Thesis, George Washington University, 1998.

[15] KAUFFMAN, L. H. and S. LAMBROPOULOU. On the classification of rational tangles. To appear in *Adv. in Appl. Math.*

[16] KAWAUCHI, A. *A Survey of Knot Theory.* Birkhäuser Verlag, 1996.

[17] KHINCHIN, A. YA. *Continued Fractions.* Dover, 1997. (Republication of the 1964 edition of Chicago Univ. Press.)

[18] KOLDEN, K. Continued fractions and linear substitutions. *Arch. Math. Naturvid. 6* (1949), 141–196.

[19] LICKORISH, W. B. R. *An Introduction to Knot Theory.* Springer Graduate Texts in Mathematics *175*, 1997.

[20] MENASCO, W. and M. THISTLETHWAITE. The classification of alternating links. *Ann. of Math. (2) 138* (1993), 113–171.

[21] MONTESINOS, J. M. Revêtements ramifiés des nœuds, espaces fibrés de Seifert et scindements de Heegaard. *Publicaciones del Seminario Matemático García de Galdeano, Serie II, Sección 3* (1984).

[22] MURASUGI, K. *Knot Theory and its Applications.* (Translated from the 1993 Japanese original by B. Kurpita.) Birkhäuser Verlag, 1996.

[23] OLDS, C. D. *Continued Fractions.* New Mathematical Library *9*, Math. Assoc. of America, 1963.

[24] PRASOLOV, V. V. and A. B. SOSSINSKY. *Knots, Links, Braids and 3-Manifolds.* AMS Translations of Mathematical Monographs *154*, 1997.

[25] PRZYTYCKI, J. H. and A. YASUHARA. Symmetry of links and classification of lens spaces. Preprint, arXiv: math.GT/0011119, 17 Nov 2000.

[26] REIDEMEISTER, K. *Knotentheorie.* (Reprint.) Chelsea, New York, 1948.

[27] —— Knoten und Verkettungen. *Math. Z. 29* (1929), 713–729.

[28] —— Homotopieringe und Linsenräume. *Abh. Math. Sem. Hansischen Univ. 11* (1936), 102–109.

[29] ROLFSEN, D. *Knots and Links.* Publish or Perish Press, Berkeley, 1976.

[30] SCHUBERT, H. Über eine numerische Knoteninvariante. *Math. Z. 61* (1954), 245–288.

[31] —— Knoten mit zwei Brücken. *Math. Z. 65* (1956), 133–170.

[32] SEIFERT, H. und W. THRELFALL. *Lehrbuch der Topologie.* Leipzig, 1934; Chelsea, 1947.

[33] SEIFERT, H. Die Verschlingungsinvarianten der zyklischen Knotenüberlagerungen. *Abh. Math. Sem. Hamburg 11* (1936), 84–101.

[34] SAWOLLEK, J. Tait's flyping conjecture for 4-regular graphs. Preprint, 1998.

[35] SIEBENMANN, L. *Lecture Notes on Rational Tangles.* Orsay, 1972 (unpublished).

[36] SIMONY, O. Über eine Reihe neuer Thatsachen aus dem Gebiete der Topologie. *Math. Ann. 19* (1882), 110–120.

[37] —— Über eine Reihe neuer Thatsachen aus dem Gebiete der Topologie, Teil II. *Math. Ann. 24* (1884), 253–280.

[38] —— Demonstration eines neuen topologischen Apparates zur Herstellung gewisser, zahlentheoretisch verwerthbarer Knotensysteme. *Deutsch. Natf. Tagebl.* (1887), 110–111.

[39] —— Über den Zusammenhang gewisser topologischer Thatsachen mit neuen Sätzen der höheren Arithmetik und dessen theoretische Bedeutung [1887]. *Wien, Ak. Sber 96* (1888), (Abth.2), 191–286; *Deutsch. Natf. Tagebl.* (1887), 229–231.

[40] SUMNERS, D. W. Untangling DNA. *Math.Intelligencer 12* (1990), 71–80.

[41] SUNDBERG, C. and M. THISTLETHWAITE. The rate of growth of the number of alternating links and tangles. *Pacific J. Math. 182* (1998), 329–358.

[42] TAIT, P. G. On knots I. II. III. In: *Scientific Papers I*, 273–437 (1877–1885). Cambridge Univ. Press, London, 1898.

[43] TIETZE, H. Über verknotete und verkettete Linien I. Über die speziellen Simony-Knoten und Simony-Ketten. *S.-B. Math.-Nat. Abt. Bayer. Akad. Wiss.* (1942), 53–62.

[44] —— Bemerkungen über verknotete und verkettete Linien II. Vorgeschriebene singuläre Primzahlen für eine Simony-Figur und für ihr Spiegelbild. *S.-B. Math.-Nat. Abt. Bayer. Akad. Wiss.* (1943), 265–268.

[45] —— Über spezielle Simony-Knoten und Simony-Ketten mit vorgeschriebenen singulären Primzahlen I, II. *Monatshefte für Math. und Phys. 51* (1943–44), 1–14; 85–100.

[46] —— Über spezielle Simony-Knoten und Simony-Ketten mit vorgeschriebenen singulären Primzahlen für die Figur und für ihr Spiegelbild. *Math. Z. 49* (1943–44), 351–369.

[47] WALL, H. S. *Analytic Theory of Continued Fractions.* Van Nostrand Company, 1948.

(Reçu le 22 novembre 2002)

L.H. Kauffman

Department of Mathematics, Statistics and Computer Science
University of Illinois at Chicago
851 South Morgan St.
Chicago, IL 60607-7045
U. S. A.
kauffman@math.uic.edu

S. Lambropoulou

Department of Mathematics
National Technical University of Athens
Zografou Campus
GR-157 80 Athens
Greece
sofia@math.ntua.gr