From this we deduce

THEOREM 1.14 ([EG2, BEG, FeV], conjectured in [FV]). *The ring Q_m of m-quasi-invariants is Gorenstein.*

Proof. By Stanley's theorem (see [Eis]), a positively graded Cohen-Macaulay domain A is Gorenstein iff its Poincaré series is a rational function $h(t)$ satisfying the equation $h(t^{-1}) = (-1)^n t^l h(t)$, where l is an integer and n is the dimension of the spectrum of A. Thus the result follows immediately from Proposition 1.13. □

1.6 THE RING OF DIFFERENTIAL OPERATORS ON X_m

Finally, let us introduce the ring $\mathcal{D}(X_m)$ of differential operators on X_m, that is the ring of differential operators with coefficients in $\mathbf{C}(\mathfrak{h})$ mapping Q_m to Q_m. It is clear that this definition coincides with Grothendieck's well-known definition ([Bj]).

THEOREM 1.15 ([BEG]). *$\mathcal{D}(X_m)$ is a simple algebra.*

REMARK 1.16. a) The ring of differential operators on a smooth affine algebraic variety is always simple (see [Bj], Chapter 3).

b) By a result of M. van den Bergh [VdB], for a non-smooth variety, the simplicity of the ring of differential operators implies the Cohen-Macaulay property of this variety.

2. LECTURE 2

We will now see how the ring Q_m appears in the theory of completely integrable systems.

2.1 HAMILTONIAN MECHANICS AND INTEGRABLE SYSTEMS

Recall the basic setup of Hamiltonian mechanics [Ar]. Consider a mechanical system with configuration space X (a smooth manifold). Then the phase space of this system is T^*X, the cotangent bundle on X. The space T^*X is naturally a symplectic manifold, and in particular we have an operation of Poisson bracket on functions on T^*X. A point of T^*X is a pair (x,p), where $x \in X$ is the position and $p \in T_x^*X$ is the momentum. Such pairs are