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EXAMPLE 3.15. If  $\lambda = 0$  and  $\tau = \mathbf{1}$  is the trivial representation of  $W$ , the Verma module  $M(0, \mathbf{1}) = \mathbf{C}[\mathfrak{h}]$ . The action of  $\mathbf{C}[\mathfrak{h}]$  is given by multiplication, that of  $\mathbf{C}[\mathfrak{h}^*]$  is generated by the Dunkl operators and  $W$  acts in the usual way.

### 3.7 GENERIC $c$

Opdam and Rouquier have recently studied the structure of the categories  $\mathcal{O}(H_c)$ ,  $\mathcal{O}(eH_c e)$ , and found that it is especially simple if  $c$  is “generic” in a certain sense. Namely, recall that for a  $W$ -invariant function  $q: \Sigma \rightarrow \mathbf{C}^*$  one can define the *Hecke algebra*  $\text{He}_q(W)$  to be the quotient of the group algebra of the fundamental group of  $U/W$  by the relations  $(T_s - 1)(T_s + q_s) = 0$ , where  $T_s$  is the image in  $U/W$  of a small half-circle around the hyperplane of  $s$  in the counterclockwise direction. It is well known that  $\text{He}_q(W)$  is an algebra of dimension  $|W|$ , which coincides with  $\mathbf{C}[W]$  if  $q = 1$ . It is also known that  $\text{He}_q(W)$  is semisimple (and isomorphic to  $\mathbf{C}[W]$  as an algebra) unless  $q_s$  belongs for some  $s$  to a finite set of roots of unity depending on  $W$  (see [Hu]).

DEFINITION 3.16. The function  $c$  is said to be *generic* if for  $q = e^{2\pi ic}$ , the Hecke algebra  $\text{He}_q(W)$  is semisimple.

In particular, any irrational  $c$  is generic, and (more important for us) an integer valued  $c$  is generic (since in this case  $q = 1$ ). We can now state the following central result:

THEOREM 3.17 (Opdam-Rouquier [OR]; see also [BEG] for an exposition). *If  $c$  is generic (in particular, if  $c$  takes non negative integer values), then the irreducible objects in  $\mathcal{O}$  are exactly the modules  $M(\lambda, \tau)$ . Moreover, the category  $\mathcal{O}$  is semisimple.*

We also have

THEOREM 3.18 ([OR]). *If  $c$  is generic then the functor  $F$  is an equivalence of categories.*

From Theorem 3.17 we can deduce

THEOREM 3.19 ([BEG]). *If  $c$  is generic, then  $H_c$  is a simple algebra.*

In the case  $c = 0$ , we get the simplicity of  $\mathbf{C}[\mathfrak{h} \oplus \mathfrak{h}^*] \rtimes \mathbf{C}[W]$ , which is well known.