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connected component by  $SO(3)$  (i.e. by “any” group of the same rank), or the group of components by  $\mathbf{Z}/4$  (i.e. by “any” group of the same size), will force the extension to be split.

*Proof.* The assertions about the connected component and the group of components are clear. Let us show that the extension associated to  $G$  is not split. Let us denote  $[g, \gamma] \in G$  the image of  $(g, \gamma) \in SU(2) \times D_8$  under the canonical projection. Let  $\mathbf{S}^1$  denote the standard maximal torus in  $SU(2)$ , and let  $N$  denote its normalizer in  $G$ . We have

$$N = \{[t, e] : t \in \mathbf{S}^1\} \amalg \{[jt, e] : t \in \mathbf{S}^1\} \amalg \{[t, r] : t \in \mathbf{S}^1\} \amalg \{[jt, r] : t \in \mathbf{S}^1\} \\ \amalg \{[t, s] : t \in \mathbf{S}^1\} \amalg \{[jt, s] : t \in \mathbf{S}^1\} \amalg \{[t, rs] : t \in \mathbf{S}^1\} \amalg \{[jt, rs] : t \in \mathbf{S}^1\}.$$

By contradiction, suppose that the extension associated to  $G$  is split, i.e. there exists a section. As  $\mathbf{Z}/2 \times \mathbf{Z}/2$  is abelian, thus nilpotent, we deduce, by Proposition 5.4, that the extension associated to  $N$  is also split. We want to show that this is not possible by considering the elements of order 2 in  $N$ . For  $n = 0, 1$ , a straightforward calculation shows that in the component corresponding to  $r^n s$ , an element  $[t, r^n s]$  is of order 2 if and only if  $t = \pm 1$ , and that the sub-component  $\{[jt, r^n s] : t \in \mathbf{S}^1\}$  does not contain any element of order 2. Two of the three non-trivial elements in  $\Gamma \cong \mathbf{Z}/2 \times \mathbf{Z}/2$  must thus be mapped by the section to  $[\pm 1, s]$  and  $[\pm 1, rs]$ . Therefore, as the section is a homomorphism, the image of the third non-trivial element is

$$[\pm 1, rs] \cdot [\pm 1, s] = [\pm 1, r],$$

which is not of order 2. A contradiction that shows that the extension associated to  $G$  is not split.

The property of minimality follows by Theorem 5.3, and by the fact that any extension with  $SO(3)$  as normal subgroup is a direct product (because  $SO(3)$  is complete, i.e. centerless and with trivial outer automorphism group).  $\square$

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