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TILE HOMOTOPY GROUPS

by Michael REID

ABSTRACT. The technique of using checkerboard colorings to show the impossibility of some tiling problems is well-known. Conway and Lagarias have introduced a new technique using boundary words. They show that their method is at least as strong as any generalized coloring argument. They successfully apply their technique, which involves some understanding of specific finitely presented groups, to two tiling problems. Partly because of the difficulty in working with finitely presented groups, their technique has only been applied in a handful of cases.

We present a slightly different approach to the Conway-Lagarias technique, which we hope provides further insight. We also give a strategy for working with the finitely presented groups that arise, and we are able to apply it in a number of cases.

1. INTRODUCTION

A classical problem is the following (see [3, pp. 142, 394], [7]).

Remove two diagonally opposite corners from a checkerboard. Dominoes are placed on the board, each covering exactly two (vertically or horizontally) adjacent squares. Can all 62 *squares be covered by* 31 *dominoes ?*

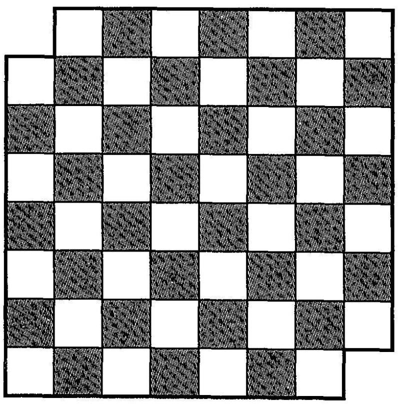

FIGURE 1.1
Mutilated checkerboard