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We may now define  $|\xi|_h^2$  for  $\xi \in L_x$  with  $x \in X$  by

$$|\xi|_h^2 = \begin{cases} e^{-\gamma(x)} |\xi/s(x)|^2 & \text{if } x \in X \setminus Y, \\ e^{-\alpha(x) - \epsilon\beta(x)} |\xi|_k^2 & \text{if } x \in V. \end{cases}$$

Then  $h$  is a well-defined  $C^\infty$  Hermitian metric in  $L$  since, for  $x \in V \setminus Y$  and  $\xi \in L_x$ , we have

$$e^{-\gamma(x)} |\xi/s(x)|^2 = e^{-(\alpha(x) - \log |s(x)|_k^2 + \epsilon\beta(x))} |\xi|_k^2 / |s(x)|_k^2 = e^{-\alpha(x) - \epsilon\beta(x)} |\xi|_k^2.$$

Furthermore, on  $X \setminus Y$  we have

$$\mathcal{R}_h = \Delta_g(-\log |s|_h^2) = \Delta_g \gamma \begin{cases} > 0 & \text{on } X \setminus Y \\ > 1 & \text{on } V \setminus Y \end{cases}$$

By continuity, we also have  $\mathcal{R}_h \geq 1 > 0$  at points in  $Y$ . Thus  $\mathcal{R}_h > 0$  on  $X$ .  $\square$

For  $X$  a Riemann surface, the above proofs become especially simple. For example, the construction of  $\alpha$  in the proof of Theorem 2.3 is trivial for  $\dim X = 1$  because  $Y$  is discrete. For  $X$  an open Riemann surface, Theorem 0.1 provides a  $C^\infty$  strictly plurisubharmonic exhaustion function and, therefore, by [Gr] and [DG], one gets the theorem of [BS] that an open Riemann surface is Stein. For a compact Riemann surface  $X$ , Theorem 2.3 becomes the familiar fact (see, for example, [GriH]) that the holomorphic line bundle associated to a nontrivial effective divisor admits a  $C^\infty$  Hermitian metric  $h$  with positive curvature  $\Theta_h$ .

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