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Autor: Ovsienko, Valentin

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where (F_i, G_j) in a $2n$ -tuple of smooth functions on M . The space $\text{TVect}(M)$ is now identified with the direct sum

$$\text{TVect}(M) \cong \underbrace{C^\infty(M) \oplus \cdots \oplus C^\infty(M)}_{2n \text{ times}}.$$

Let us calculate explicitly the action of $\text{CVect}(M)$ on $\text{TVect}(M)$.

PROPOSITION 5.4. *The action of $\text{CVect}(M)$ on $\text{TVect}(M)$ is given by the first-order $(2n \times 2n)$ -matrix differential operator*

$$(14) \quad X_H \begin{pmatrix} F \\ G \end{pmatrix} = \left(X_H \cdot \mathbf{1} - \begin{pmatrix} AB(H) & BB(H) \\ -AA(H) & -BA(H) \end{pmatrix} \right) \begin{pmatrix} F \\ G \end{pmatrix},$$

where F and G are n -vector functions, $\mathbf{1}$ is the unit $(2n \times 2n)$ -matrix, $AA(H)$, $AB(H)$, $BA(H)$ and $BB(H)$ are $(n \times n)$ -matrices, namely

$$AA(H)_{ij} = A_i A_j(H),$$

and the three other expressions are similar.

Proof. Straightforward from (11) and (13). \square

PROPOSITION 5.5. *The bilinear map (7) has the following explicit expression:*

$$H_{X, \tilde{X}} = \sum_{i=1}^n \begin{vmatrix} F_i & \tilde{F}_i \\ G_i & \tilde{G}_i \end{vmatrix},$$

where $X = \sum_{i=1}^n (F_i A_i + G_i B_i)$, and $\tilde{X} = \sum_{j=1}^n (\tilde{F}_j A_j + \tilde{G}_j B_j)$.

Proof. This follows from definition (7) and formula (12). \square

Note that formula (14) implies that $H_{X, \tilde{X}}$ transforms as a contact Hamiltonian according to (3) since the partial traces of the $(2n \times 2n)$ -matrix in (14) are $A_i B_i(H) - B_i A_i(H) = Z(H)$.

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V. Ovsienko

CNRS, Institut Camille Jordan
Université Claude Bernard Lyon 1
21, avenue Claude Bernard
F-69622 Villeurbanne Cedex
France
e-mail: ovsienko@math.univ-lyon1.fr

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