

Zeitschrift: L'Enseignement Mathématique

Band: 54 (2008)

Heft: 1-2

Artikel: Soluble groups of type VF

Autor: Martínez-Pérez, Conchita / Nucinkis, Brita E. A.

DOI: <https://doi.org/10.5169/seals-109920>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 07.10.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

51

SOLUBLE GROUPS OF TYPE VF

by Conchita MARTÍNEZ-PÉREZ and Brita E. A. NUCINKIS

Cohomological finiteness conditions for soluble groups are very well understood for torsion-free soluble groups, but there remains a gap for soluble groups with torsion. This can be summarised by the following conjecture:

CONJECTURE 51.1. *Every soluble group G of type VF admits a cocompact model for \underline{EG} .*

A group is said to be *of type VF* if it has a finite index subgroup admitting a finite $K(G, 1)$.

Finiteness conditions for soluble groups have been attracting wide attention ever since the celebrated result by Stammbach [9], that for a torsion-free soluble group the homological dimension $\text{hd } G$ is equal to the Hirsch length $\text{h } G$ of the group. For polycyclic groups the Hirsch length is equal to the cohomological dimension, $\text{cd } G$. (For countable groups, the homological dimension and the cohomological dimension differ by at most one.)

This result was extended by Gruenberg, see [1], who showed that a torsion-free nilpotent group is finitely generated if and only if $\text{cd } G = \text{h } G$. This led to the question, which cohomological finiteness condition describes torsion-free soluble groups with $\text{cd } G = \text{h } G$.

There were partial answers to this question by numerous authors including Bieri, Gildenhuys and Strebel and it was finally answered by Kropholler [3]. Kropholler's later result [4] meant that this result could be phrased as follows:

THEOREM 51.2 ([3,4]). *Let G be a soluble group. Then the following are equivalent:*

- (1) G is of type FP_∞ ,
- (2) G is virtually of type FP ,
- (3) $\text{vcd } G = \text{h } G < \infty$,
- (4) G is virtually torsion-free and constructible.

The fact that G is constructible implies that G is finitely presented. Thus any torsion-free group satisfying the conditions of the Theorem is of type F , i.e. it has a finite $\text{K}(G, 1)$ or equivalently a cocompact model for EG .

In case G has torsion it cannot admit a finite dimensional model for EG , which is equivalent to saying that $\text{cd } G = \infty$. We say a G -CW-complex X is a model for $\underline{\text{EG}}$ if X^H is contractible for all finite $H < G$ and empty otherwise. The cohomological counterpart is Bredon (co)homology. It was shown [8] that for countable groups the Bredon cohomological dimension $\underline{\text{cd}} G$ and the Bredon homological dimension $\underline{\text{hd}} G$ differ by at most one. By using a spectral sequence of Martínez-Pérez [7], Flores and Nucinkis [2] proved the analogue to Stambach's result, namely that for soluble groups, $\underline{\text{hd}} G = \text{h } G$. This led us to pose the following conjecture:

CONJECTURE 51.3. *Let G be a soluble group. Then the following are equivalent:*

- (1) G is of type $\underline{\text{FP}}_\infty$,
- (2) $\underline{\text{cd}} G = \text{h } G < \infty$,
- (3) G is of type FP_∞ .

$\underline{\text{FP}}_\infty$ denotes the Bredon analogue to FP_∞ . It is not hard to see that (1) \Rightarrow (2) \Rightarrow (3), see [2]. There are, however, examples by Leary and Nucinkis [5] showing that generally groups of type VF do not necessarily admit a cocompact model for $\underline{\text{EG}}$, but all available evidence leads us to believe that Conjecture 51.1 still holds for soluble groups. Lück [6] showed that a group admits a model of finite type for $\underline{\text{EG}}$ if and only if it is finitely presented of type FP_∞ , has finitely many conjugacy classes of finite subgroups and all centralisers of finite subgroups are of type FP_∞ . But even with this reduction, an answer to both conjectures remains frustratingly elusive.

REFERENCES

- [1] BIERI, R. *Homological Dimension of Discrete Groups*. Queen Mary College Mathematics Notes, London, 1976.
- [2] FLORES R.J. and B.E.A. NUCINKIS. On Bredon homology of elementary amenable groups. *Proc. Amer. Math. Soc.* 135 (2007), 5–11.
- [3] KROPHOLLER, P.H. Cohomological dimensions of soluble groups. *J. Pure Appl. Algebra* 43 (1986), 281–287.
- [4] ——— On groups of type FP_∞ . *J. Pure Appl. Algebra* 90 (1993), 55–67.
- [5] LEARY, I.J. and B.E.A. NUCINKIS. Some groups of type VF. *Invent. Math.* 151 (2003), 135–165.
- [6] LÜCK, W. The type of the classifying space for a family of subgroups. *J. Pure Appl. Algebra* 149 (2000), 177–203.
- [7] MARTÍNEZ-PÉREZ, C. A spectral sequence in Bredon (co)homology. *J. Pure Appl. Algebra* 176 (2002), 161–173.
- [8] NUCINKIS, B.E.A. On dimensions in Bredon homology. *Homology Homotopy Appl.* 6 (2004), 33–47.
- [9] STAMMBACH, U. On the weak homological dimension of the group algebra of solvable groups. *J. London Math. Soc. (2)* 2 (1970), 567–570.

Conchita Martínez-Pérez

Universidad de Zaragoza
 E-50009 Zaragoza
 Spain
e-mail: conmar@unizar.es

Brita E.A. Nucinkis

University of Southampton
 Southampton SO17 1BJ
 England
e-mail: B.E.A.Nucinkis@soton.ac.uk