

Zeitschrift: L'Enseignement Mathématique
Band: 54 (2008)
Heft: 1-2

Artikel: Residually nilpotent group
Autor: Mikhailov, Roman / Passi, Inder Bir S.
DOI: <https://doi.org/10.5169/seals-109921>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 07.10.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

52

RESIDUALLY NILPOTENT GROUPS

by Roman MIKHAILOV and Inder Bir S. PASSI

In general, it is difficult to decide whether a given group is residually nilpotent; this is so even for rather simple looking one-relator groups [2]. There is thus need to develop general methods for checking residual nilpotence. Such an investigation will have wide impact in group theory and topology; for example, in the context of Baumslag's parafree conjecture [1], Whitehead's asphericity conjecture [4], etc.

We list here two problems on residual nilpotence (see also Kourovka Notebook 2006, Problem 16.65).

PROBLEM 52.1. If F is a free group with finite basis x_1, \dots, x_n , and r a basic commutator, then is the group $\langle x_1, \dots, x_n \mid r \rangle$ residually nilpotent?

Let G be a residually nilpotent group. We say that G is *absolutely residually nilpotent* if for any k -central extension

$$1 \rightarrow N \rightarrow \tilde{G} \rightarrow G \rightarrow 1,$$

of G , i.e., a central extension satisfying $[N, \underbrace{\tilde{G}, \dots, \tilde{G}}_{k \text{ terms}}] = 1$, the group \tilde{G} is again residually nilpotent.

It would be of interest to investigate such groups; for instance, it can be shown that the following two statements are equivalent:

- (i) finitely-generated parafree groups are absolutely residually nilpotent;
- (ii) Baumslag's parafree conjecture: $H_2(G) = 0$ for a finitely-generated parafree group G .

As a first step, on examining absolute residual nilpotence for one-relator groups, it turns out that *every central extension of a one-relator residually*

nilpotent group is again residually nilpotent [3]. We are thus motivated to raise the following:

PROBLEM 52.2. Is every one-relator residually nilpotent group absolutely residually nilpotent?

REFERENCES

- [1] COCHRAN, T.D. and K. E. ORR. Stability of lower central series of compact 3-manifold groups. *Topology* 37 (1998), 497–526.
- [2] MAGNUS, W., A. KARRASS and D. SOLITAR. *Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations*. Pure and Applied Mathematics 13. Interscience Publishers, New York, 1966.
- [3] MIKHAILOV, R. Residual nilpotence and residual solubility of groups. *Sb. Math.* 196 (2005), 1659–1675.
- [4] WHITEHEAD, J.H.C. On adding relations to homotopy groups. *Ann. of Math.* 42 (1941), 409–428.

Roman Mikhailov

Steklov Mathematical Institute
Department of Algebra
Gubkina 8
Moscow, 119991
Russia
e-mail: rmikhailov@mail.ru

Inder Bir S. Passi

Centre for Advanced Study
in Mathematics
Panjab University
Chandigarh, 160041
India
e-mail: ibspassi@yahoo.co.in