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# 52

## RESIDUALLY NILPOTENT GROUPS

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In general, it is difficult to decide whether a given group is residually nilpotent; this is so even for rather simple looking one-relator groups [2]. There is thus need to develop general methods for checking residual nilpotence. Such an investigation will have wide impact in group theory and topology; for example, in the context of Baumslag's parafree conjecture [1], Whitehead's asphericity conjecture [4], etc.

We list here two problems on residual nilpotence (see also Kourovka Notebook 2006, Problem 16.65).

PROBLEM 52.1. If $F$ is a free group with finite basis $x_1, \dots, x_n$, and $r$ a basic commutator, then is the group $\langle x_1, \dots, x_n \mid r \rangle$ residually nilpotent?

Let $G$ be a residually nilpotent group. We say that $G$ is *absolutely residually nilpotent* if for any $k$-central extension

$$1 \to N \to \widetilde{G} \to G \to 1,$$

of $G$, i.e., a central extension satisfying $[N, \underbrace{\widetilde{G}, \dots, \widetilde{G}}_{k \text{ terms}}] = 1$, the group $\widetilde{G}$ is again residually nilpotent.

It would be of interest to investigate such groups; for instance, it can be shown that the following two statements are equivalent:

(i) finitely-generated parafree groups are absolutely residually nilpotent;

(ii) Baumslag's parafree conjecture: $H_2(G) = 0$ for a finitely-generated parafree group $G$.

As a first step, on examining absolute residual nilpotence for one-relator groups, it turns out that *every central extension of a one-relator residually* *nilpotent group is again residually nilpotent* [3]. We are thus motivated to raise the following:

PROBLEM 52.2. Is every one-relator residually nilpotent group absolutely residually nilpotent?

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