

A realization problem

Autor(en): **Varadarajan, Kalathoor**

Objektyp: **Article**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **54 (2008)**

Heft 1-2

PDF erstellt am: **23.07.2024**

Persistenter Link: <https://doi.org/10.5169/seals-109932>

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.

Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.

Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

63

A REALIZATION PROBLEM

by Kalathoor VARADARAJAN

In his groundbreaking papers ([7], [8]) C. T. C. Wall associated with each (always assumed 0-connected) finitely dominated space X an element $\tilde{w}(X)$ in $\widetilde{K}_0(\mathbf{Z}\pi)$ where $\pi = \pi_1(X)$ and proved that X is of the homotopy type of a finite CW-complex if and only if $\tilde{w}(X) = 0$. Also $\tilde{w}(X)$ is an invariant of the homotopy type of X . In subsequent literature $\tilde{w}(X)$ is referred to as the finiteness obstruction (alternatively as the Wall obstruction) of X . Another major result proved by Wall asserts that given any finitely presented group π and any element x in $\widetilde{K}_0(\mathbf{Z}\pi)$, there exists a finitely dominated CW-complex X with $\pi_1(X)$ isomorphic to π and $\tilde{w}(X) = x$. Using Dock Sang Rim's result [5] that $\widetilde{K}_0(\mathbf{Z}\pi_p)$ for any prime p is isomorphic to the ideal class group $\text{Cl}(\mathbf{Z}[\omega])$, where π_p denotes a cyclic group of order p and $\omega = \exp(\frac{2\pi i}{p})$ and the fact that $\text{Cl}(\mathbf{Z}[\omega])$ is not zero when $p = 23$, Wall shows that there exist finitely dominated CW-complexes which are not of the homotopy type of a finite CW-complex. This settled a famous problem of J. H. C. Whitehead [9] in the negative.

Guido is the first person who started studying the Wall obstruction of finitely dominated nilpotent spaces [2] and [3]. In his 1976 work he proved that $\tilde{w}(X) = 0$ for any finitely dominated nilpotent space with $\pi_1(X)$ infinite. In his 1975 work he showed that if X is a finitely dominated nilpotent space with $\pi_1(X)$ finite cyclic, then $\tilde{w}(X)$ has to satisfy certain restrictions. Inspired by his results, I extended his 1975 results to finitely dominated nilpotent spaces with finite abelian fundamental groups. My result [6] appeared in 1978. For any nilpotent group π , let $\overline{\mathbf{Z}\pi}$ denote a maximal order in $\mathbf{Q}\pi$ containing $\mathbf{Z}\pi$ and $D(\mathbf{Z}\pi)$ denote the kernel of

$$j_* : \widetilde{K}_0(\mathbf{Z}\pi) \rightarrow \widetilde{K}_0(\overline{\mathbf{Z}\pi}).$$

In the joint paper [4] in 1979, Guido and myself showed that for any finitely dominated nilpotent space X with a finite (necessarily nilpotent) fundamental

group π , the Wall obstruction $\tilde{w}(X)$ satisfies the restriction that $\tilde{w}(X)$ is in $D(\mathbf{Z}\pi)$. This considerably strengthened the result in [6].

As stated earlier in this article, for any finitely presented group π and any element x in $\widetilde{K}_0(\mathbf{Z}\pi)$, there exists a finitely dominated CW-complex X with $\pi_1(X) = \pi$ and $\tilde{w}(X) = x$ (Wall's work in 1965, 1966). In [1], Ewing, Löffler and Pedersen showed that for a finite nilpotent group of composite order, the set of elements of $\widetilde{K}_0(\mathbf{Z}\pi)$ that can be realized as the finiteness obstruction of a nilpotent space with fundamental group π is not in general equal to $D(\mathbf{Z}\pi)$. This suggests the following.

QUESTION 63.1. *Given a finite nilpotent group π characterize completely the elements in $D(\mathbf{Z}\pi)$ which occur as the finiteness obstruction of a finitely dominated nilpotent space and for such an element x give an explicit construction of a finitely dominated nilpotent space X with $\tilde{w}(X) = x$.*

In this article, I have concentrated on just one aspect of Guido's work. His work is very profound and has influenced the development of topology in many ways.

REFERENCES

- [1] EWING, J., P. LÖFFLER and E. K. PEDERSEN. A local approach to the finiteness obstruction. *Quart. J. Math. Oxford Ser. (2)* 39 (1988), 443–461.
- [2] MISLIN, G. Wall's obstruction for nilpotent spaces. *Topology* 14 (1975), 311–317.
- [3] ———. Finitely dominated nilpotent spaces. *Ann. of Math.* 103 (1976), 547–556.
- [4] MISLIN, G. and K. VARADARAJAN. The finiteness obstructions for nilpotent spaces lie in $D(\mathbf{Z}\pi)$. *Invent. Math.* 53 (1979), 185–191.
- [5] RIM, D. S. Modules over finite groups. *Ann. of Math.* 69 (1959), 700–712.
- [6] VARADARAJAN, K. Finiteness obstruction for nilpotent spaces. *J. Pure Appl. Algebra* 12 (1978), 137–146.
- [7] WALL, C. T. C. Finiteness conditions for CW-complexes. *Ann. of Math. (2)* 81 (1965), 56–69.
- [8] ———. Finiteness conditions for CW-complexes. II. *Proc. Roy. Soc. London Ser. A* 295 (1966), 129–139.
- [9] WHITEHEAD, J. H. C. Combinatorial Homotopy I and II. *Bull. Amer. Math. Soc.* 55 (1949), 213–245 and 453–496.

K. Varadarajan

Department of Mathematics and Statistics
 University of Calgary
 Calgary, Alberta, T2N1N4
 Canada
e-mail: varadara@math.ucalgary.ca