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Cosmological Theory

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Cosmology, in the broadest sense of the word, is that branch of learning which treats of the Universe as an ordered system. Of recent years cosmology has come to deal more particularly with the study of the distribution in position and motion of matter and energy in the large, seeking out those general traits which characterize the nebular universe and exploring their implications for the past and for the future. This trend in cosmology can be attributed to two, initially quite separate, developments of the present century — the formulation of the relativity theories on the one hand, and the enormous widening of our astronomical horizon made possible by the great telescopes and their ancillary tools. The confluence of these two streams within the past few decades has resulted in an ordered picture of the universe as a whole which, although it may not as yet have given us an unequivocal quantitative model, nevertheless serves as a challenge to theory and as a guide to observation.

In this brief account of cosmological theory I shall diverge from the path of the historical development, presenting only a skeletal framework which seems to me to encompass the broad achievements of recent years and which may serve as a platform from which to launch advances in the future. The elements of this framework are the contributions of many of our past and present colleagues, but among them stand out above all those of EINSTEIN on the theoretical, more mathematical, side, and of HUBBLE on the observational, more astronomical, aspects. I am sure it is to all of you, as it is to me, a source of great regret and even of personal loss that these two masters cannot be with us today to express their own views on this subject to which they each contributed so much.

In keeping with this program, I shall in Part I of my talk sketch briefly the mathematical framework which is available for – I might even say is forced upon – any cosmological theory which treats the universe as a spatially uniform continuum, at each event of which the mean motion of matter is essentially unique. These assumed uniformities in the substructure imply the existence of a universal or 'cosmic' time t, and a layering of space-time into homogenous and isotropic spaces $t =$ const., the whole being necessarily tied together by a quadratic metric adapted to a pertinent description of the material and energetic content. This kinematical model of the universe is determined in principle by the curvature $K(t)$ of the various spatial sections – or more precisely, by the sign k of the curvature and a single function $R(t)$ which, for $k \neq 0$, is the equivalent 'radius' defined by $K(t) = k/R^2(t)$.

In Part II, I turn to the problem of interpreting the physical observables – such as apparent magnitude, apparent diameter and redshift – in terms of the abstract elements of the mathematical framework – the variables representing distance, size and velocity. The relations which exist between these latter mathematical concepts, in virtue of the geometry and kinematics prescribed in the model, can be translated into relations between the corresponding observables. The appeal to the empirical should then give certain limited information concerning the present value and the present trends of the function $R(t)$ defining the model. At the present time the most promising empirical approach appears to lie in the determination of the relation between the redshift $z = \Delta\lambda/\lambda$ and the apparent magnitude m of distant nebulae. For this examination I am very pleased to be able to present here the results of a survey of all existing redshift data – the first comprehensive one in twenty years – which has recently been completed by HUMASON, MAYALL and SANDAGE at the Mount Wilson-Palomar and Lick Observatories, and which the authors have most kindly put at my disposal to include in my account to you. From the deviation from the linear velocity-distance relationship, brought out by this survey, it appears that the motion of the nebulae at the present epoch is one of deceleration – a result which portends a certain amount of difficulty in reconciling the implied age of the universe with those obtained from other considerations.

But the exploitation of the observational material alone can never tell us how the nebular universe may be expected to develop in the future, nor from what it may have developed in the past. For this we must call upon field equations which relate the curvature $K(t)$ of the mathematical model to the distribution of matter and energy observed in the real world. A number of field equations which warrant attention have been proposed, including those presented in recent years by HOYLE and by JORDAN, and we may well expect that a fruitful interaction between the relativity and quantum theories will produce others more inclusive in the near future. Nervertheless I shall confine myself in the final Part III to those models whose temporal behavior is governed by the field equations of the general theory of relativity – which in spite of its impotence in dealing with MACH's Principle or the microscopic realm may yet be the springboard from which a more complete theory takes off, much as it itself took off from the Newtonian theory. Upon retaining the questionable 'cosmological constant' Λ, introduced but later disowned by EINSTEIN, we find that a unique determination of the model requires three independent empirical data. The new velocity-distance relationship offers us two such data, and as the third we could take the mean density of matter in our more immediate cosmic neighbourhood. But of these three the only one which is not beset by great uncertainty is the coefficient H of the linear term in the velocity-distance relationship; I therefore prefer to take H as the only fixed datum, and to consider the two-parameter family of general relativistic models characterized by a range of values of the present density ϱ_0 and of the epoch t_0. You may well object that t_0, the 'age of the universe', is hardly accessible to direct observation. Yet I take it as a significant parameter because other considerations require that it lie within certain reasonably well-defined limits; it must be long enough to allow for the observed geological and cosmogonical features of our local system, and yet not so long as to lead to the exhaustion of the radioactive and fusion processes taking place about us – all this, of course, on the assumption, implied in the field equations, that the total energy of the system is conserved.

The results of this survey of possible models show that we can satisfy the semi-quantitative restrictions which observation places on the density ϱ_0 and the epoch t_0 – although to do so we may have to retain the ghostly cosmological constant Λ. But even this unpalatable imposition may be avoided, for on adopting the value $H = 180$ km/s per Mpc of the Hubble constant implied by BAADE's and SANDAGE's recent revisions of the nebular distance scale, we are led to a model of the EINSTEIN-DE SITTER type ($\Lambda = 0$, $k = 0$) whose present density is $6{\cdot}2 \times 10^{-29}$ gm/cm^3 and whose age is 3·6 billion years. Although this density is a little on the high side and the age a little on the low, they are nevertheless both of an acceptable order of magnitude – and any further upward revision of the distance scale, as has been suggested by some, would improve the fit. But in spite of this, all is still not well, for the deceleration implied by the new Mount Wilson-Palomar-Lick survey requires a still greater density, and this in turn would decrease the age to a point where it would be necessary to resurrect Λ.

My general conclusion is that there is found in this examination of the cosmological problem no compelling reason for seeking an explanation of the redshift as other than DOPPLER shift due to the motion, nor for abandoning the field equations of general relativity as untenable. Nevertheless we are faced with difficulties which bid us try with open minds for new theoretical approaches, and with lacunae which urge us to augment our knowledge of the nature and disposition of matter in the large.

Part I: The Kinematical Model

The crudest model which can be expected to portray the gross characteristics of the actual universe must take account of the observed large-scale distribution, in position and motion, of the extragalactic nebulae. Allowing for their evident clustering by choosing the element of volume sufficiently large, the observations suggest that to some degree of approximation these nebulae may be considered as uniformly distributed throughout space. The only direct knowledge we have of the motions of nebulae is that obtained from the redshift in their spectra, interpreted as DOPPLER effect due to a radial component of relative motion. So far as this inferred motion is concerned, the evidence indicates that there is at each gross volume element a natural state of motion, deviations from which are small in comparison with the only significant criterion, the velocity of light.

The first step in the idealization is to replace these real nebulae by ideal average nebulae, distributed at random through observable space, and having a natural state of motion in each neighborhood. Included also must be the light rays, by which the observer obtains visual and spectrographic data on the nebulae. These traits, and these alone, being included, there should be no intrinsic characteristic involved which could serve to distinguish one nebula or one nebular region from any other; technically, the model should be homogeneous and isotropic. In dealing analytically with this model, we shall adopt the Eulerian dodge of considering its material content as a hydrodynamical fluid, rather than as particulate matter – but this is only a mathematical trick to simplify the analysis, and properly handled should have no cosmological implications.

Consider now the world-line of one such idealized nebula N, along which some cyclic 'clock' measures time t, and at each event of which light signals can be sent or received in any direction. The view of the nebular system obtained from the world-line of N must be identical with that obtained from the world-line of any other such nebula N′ – an equivalence extending even to a numerical series of clock readings, provided the clocks are intrinsically identical and are appropriately set. Further, the uniformity assumptions imply that the two-dimensional space-time surface S generated by the totality of light signals from N to N′ coincides with that generated by the signals from N′ to N, and that if one event on the world-line of a third nebula N″ is in S, then the whole of its world-line must also lie in S [1].

A signal sent out by N at time t_1, as measured by his clock, will be received by N′ at a time t'_1 which is some function $p(t_1)$ of t_1; uniformity

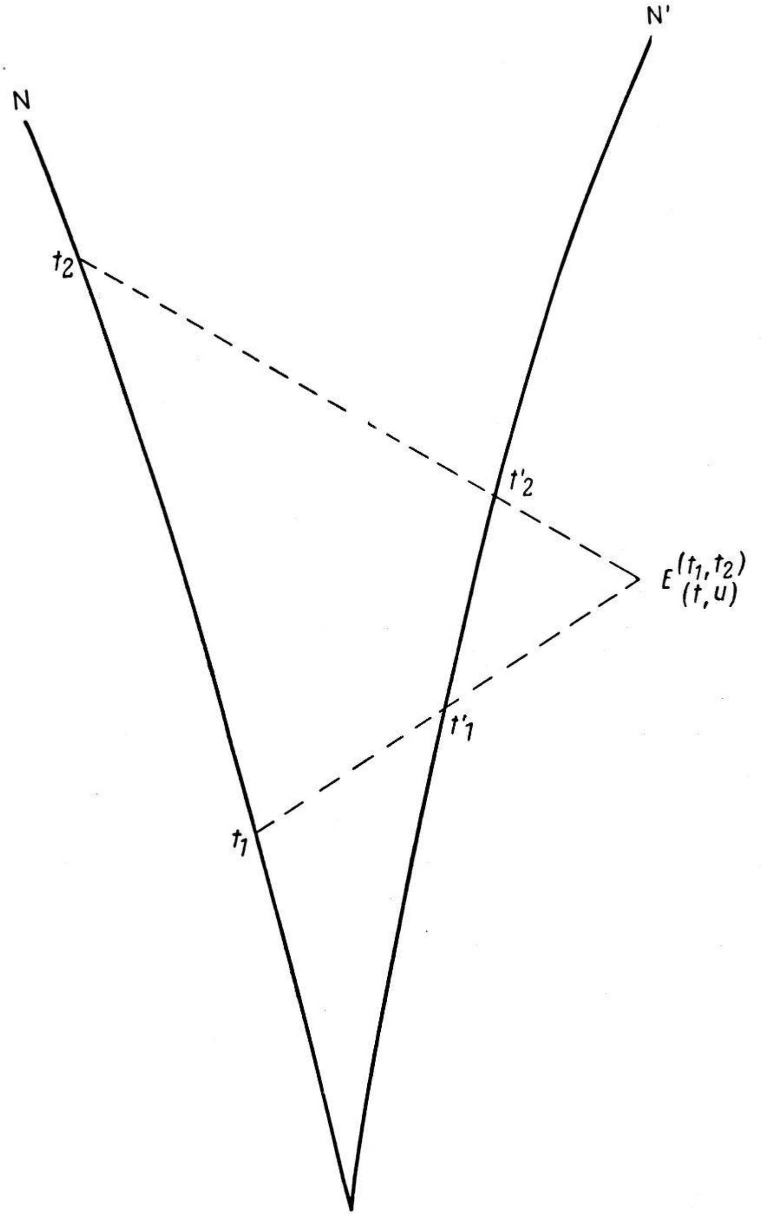

Fig. 1

then demands that a signal sent out at time t'_2 by N′ will be received by N at the time $t_2 = p(t'_2)$ defined by the same function p. The situation is as depicted in Figure 1; as there indicated, any event E in the surface S is characterized by the coordinate pair (t_1, t_2), or alternatively by the pair (t'_1, t'_2). The relations described above between these two pairs, with the aid of the function p, may be considered as the equations of transformation from the one coordinate system to the other.

Considering now the one-parameter family of world-lines lying in one such surface S, the relations

$$t_1' = p(t_1) = f(t_1, u)\,, \quad t_2 = p(t_2') = f(t_2', u) \tag{1}$$

between any two of them must constitute a one-parameter group G_1, characterized by a continuous parameter u associated with the pair N, N′. This group property gives us a very powerful tool for analysing the nature of the relations existing between any two nebulae in the surface S, and by extension between any two nebulae in the entire model. The theory of continuous groups enables us to conclude that, on appropriate normalization of the group parameter u, the relations in S are uniquely determined by a single function $\xi(t)$, the generator of the group G_1. The finite equations of the group are then

$$F(t_1') = F(t_1) + u\,, \quad F(t_2') = F(t_2) - u\,, \tag{2}$$

where

$$F(t) = \int^{t} \frac{dt}{\xi(t)}\,.$$

The parametric lines $t_1 =$ const., $t_2 =$ const. represent the two families of light rays in S, which may therefore be characterized by the vanishing of the quadratic form

$$dt_1\, dt_2 = 0\,, \quad \text{or} \quad dF(t_1)\, dF(t_2) = 0\,. \tag{3}$$

The second of these forms is invariant under the transformations of the group, for $dF(t_1)$, $dF(t_2)$ are the two fundamental differential invariants of G_1. We now ask whether an invariant form can be introduced which not only accounts for the light lines, as above, but also for the nebular world-lines themselves. Such a form depends at most on the above two differential invariants $dF(t_1)$, $dF(t_2)$ and upon the sole finite invariant $F(t_1) + F(t_2)$ of the group; in place of this later it will be found convenient to use the invariant t defined implicitly by the equation

$$F(t) = \frac{1}{2}\left[F(t_2) + F(t_1)\right]; \tag{4}$$

no inconsistency is involved in naming this invariant t, for it is in fact the same as the coordinate t for an event on the world-line $t_2 = t_1$ – a universal or 'cosmic' time which serves to synchronize those events on different world-lines from which identical world-views are obtained. A metric satisfying these conditions is of the form

$$ds^2 = \varphi(t)\, dF(t_1)\, dF(t_2) = \frac{\varphi(t)}{\xi(t_1)\,\xi(t_2)}\, dt_1\, dt_2 \,.$$

Clearly ds will measure the cosmic time interval dt along the world-line $t_1 = t_2$ provided we choose $\varphi(t) = \xi^2(t)$, i. e. choose as the metric

$$ds^2 = \frac{\xi^2(t)}{\xi(t_1)\,\xi(t_2)}\, dt_1\, dt_2 \,. \tag{5}$$

Further, the nebular world lines are geodesics of the metric ds^2, just as they are in the general theory of relativity.

On introducing as a second new coordinate the parameter

$$u = \frac{1}{2}\,[F(t_2) - F(t_1)] \tag{6}$$

arising from the group, the linear element (5) assumes in terms of t and u the Gaussian form

$$ds^2 = dt^2 - \xi^2(t)\, du^2 \,. \tag{7}$$

It is then a simple application of the HELMHOLTZ-LIE theorem to show that this quadratic linear element can be extended to the full $(3 + 1)$-dimensional space-time, where du^2 is then the metric of an auxiliary 3-dimensional space of constant Riemannian curvature – which may, on appropriate renormalization of the generator $\xi(t)$, be taken as $k = +1$, 0 or -1. Coordinates η, ϑ, φ may then be introduced in which the auxiliary metric du^2 assumes the canonical form

$$du^2 = d\eta^2 + \sigma^2(\eta)\,[d\vartheta^2 + \sin^2\vartheta\, d\varphi^2] \tag{8}$$

where $\sigma(\eta) = \sin\eta,\ \eta,\ \sinh\eta$ for $k = +1,\ 0,\ -1$, respectively.

It has thus been shown, by purely geometrical-kinematical reasoning, that the idealized nebular universe admits a quadratic metric ds^2, characterized by a single function $\xi(t)$ of cosmic time, and the sign k of the curvature of the auxiliary metric du^2. The world-lines of the idealized nebulae are the special geodesics η, ϑ, $\varphi = \text{const.}$ of ds^2, and the light-lines are the null-geodesics, exactly as in EINSTEIN's general theory of relativity. But here the existence of the metric has not been *assumed*, it has been *deduced* from the general uniformity conditions defining the problem. The choice of a specific model will of course depend on the physical theory imposed.

It is to be emphasized that we have not *required* the real universe to be one satisfying the uniformity conditions imposed above; we are merely examining the nature of that idealized model of the real world in which the obvious and all-important inhomogeneities are ironed out. We are not imposing the uniformity as a 'cosmological principle', in the terminology of MILNE [2], to which the real world must adhere.

Among the models thus found there are two which are of special interest as exhibiting the further uniformity that any event on the world-line of a nebula is intrinsically indistinguishable from any other, a situation which BONDI and GOLD [3] have characterized by the term 'perfect cosmological principle'. It is readily shown [4] that in this case we must have either the 'EINSTEIN universe'

$$\xi(t) = \text{const.}, \ k \text{ arbitrary} \tag{9 E}$$

or the 'DE SITTER universe'

$$\xi(t) = e^{t/b}, \ k = 0 \,. \tag{9 S}$$

Included as a special case of both is the familiar MINKOWSKI space-time

$$\xi(t) = \frac{1}{c^2}\,, \ k = 0 \,. \tag{9 M}$$

One further service is performed by the linear element ds^2, the measurement of spatial distance within a volume element – which is, it will be remembered, some millions of parsecs across. For locally this linear element performs the same functions as does the MINKOWSKI metric of the special theory of relativity in measuring proper time and describing the light-lines as generators of the cone $ds^2 = 0$. Hence ds^2 may also serve the same purpose of measuring local distances in the space $t = \text{const.}$; in the general coordinates here employed the spatial metric thus induced is

$$dr^2 = c^2\, \xi^2(t)\, du^2 = R^2(t)\, du^2 \tag{10}$$

where we have for later convenience written $R(t) = c\, \xi(t)$. The curvature $K(t) = k/R^2(t)$ of this metric completely defines the full cosmological model, excluding topological considerations.

Part II: Theoretical and Observational Relations

We turn now to the consideration of the implications of the kinematical model for possible observations on distant nebulae. Our knowledge of these nebulae derives solely from optical and other electromagnetic radiations which we receive from them. To examine the basic nature of this phenomenon, consider radiation which is emitted by a nebula $N(\eta, \vartheta, \varphi)$ at time t_1 and is received at time t_0 by an observer at the nebula O for which $\eta = 0$. If u be the parameter distance from O to N, as measured by the auxiliary metric du^2, then the three variables t_1, t_0 and u are tied together by the condition

$$\int_{t_1}^{t_0} \frac{dt}{\xi(t)} = u . \tag{11}$$

Next consider the light emitted by N in the interval t_1, $t_1 + dt_1$; it will be received in the interval t_0, $t_0 + dt_0$ defined by the relation

$$\frac{dt_0}{\xi(t_0)} = \frac{dt_1}{\xi(t_1)}$$

obtained from equation (11) on holding the parameter distance u between the nebulae constant. From this it follows that the change $\Delta\lambda$ in the wave-length of this light defined by the relation

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\xi_0}{\xi_1} - 1 , \tag{12}$$

where the subscripts indicate the time at which $\xi(t)$ is computed.

Holding the time t_0 of observation fixed, the first of these two equations defines the unknown time of emission t_1 in terms of the (equally unknown) parameter distance u, and the second defines the unknown t_1 in terms of the observable 'redshift' z. On eliminating the unknown t_1 between the two, we obtain the series expansion

$$z = \dot{\xi}_0 u + \frac{1}{2} (\dot{\xi}_0^2 - \xi_0 \ddot{\xi}_0) u^2 + O^3 \tag{13}$$

for the redshift in terms of u, where the dots indicate the derivatives of $\xi(t)$ with respect to its argument t. This relation expresses a dependence of the observable shift in wave-length on the parameter u, which latter is in some way a measure of the distance between the two nebulae.

To get at an approximate interpretation of this relation for the nearer nebulae, we recall the fact that for them the local MINKOWSKI distance r at the time of observation t_0 is related to u, as in equation (10), by

$$r = c\, \xi_0\, u = R_0\, u. \tag{14}$$

To the approximation in which the concepts used are valid, the principal term in the relation (13) above enables us to write

$$c\, z \sim H\, r\,, \tag{15}$$

where $H = \dot{\xi}_0/\xi_0$. Interpreting the redshift z as the DOPPLER effect due to motion of the nebula N relative to O, the term $c\,z$ on the left is to the present approximation the velocity of recession of N with respect to O, and the relation (15) expresses the approximate linear velocity-distance effect [5]. The most recent surveys, discussed more fully in the sequel, give as the value of HUBBLE's constant

$$H = 180 \text{ km/sec per megaparsec, or } 5{\cdot}9 \times 10^{-18}\ \text{sec}^{-1}. \tag{16}$$

A conceptually more satisfying interpretation of this constant is that, had the inferred nebular velocity remained the same for each nebula throughout past time, then all the nebulae in the model would have started from a common origin $1/H = 5{\cdot}4$ billion years ago.

But we have gotten ahead of the story, for the approximate distance r is not itself an observable; it is inferred in practice from the rate at which light from the nebula is received in the telescope. The true observable is thus the apparent luminosity l of the nebula, or equivalently its apparent magnitude m – the luminosity measured on a logarithmic scale. If we assume that photons are conserved in traversing internebular space, and that their energy and frequency are related by PLANCK'S law in which the constant of proportionality h is independent of cosmic time, then it can be shown that the apparent bolometric luminosity of a nebula observed at time t_0 is

$$l = \frac{L_1}{4\,\pi\, R_0^2\, \sigma^2(u)\,(1+z)^2}\,, \tag{17}$$

where L_1 is the total rate of output of energy at the time t_1 [6]. On eliminating the parameter u between equations (17) and (13), we find a relation between the observables z and m, at the expense of introducing the new parameter L_1, which is however an intrinsic property of the nebula.

This new relation, expressed in logarithmic form, has as its principal terms

$$m = M_0 - 45{\cdot}06 + 5 \log\left(\frac{z}{H}\right)$$
$$+ 1{\cdot}086\left(1 + \frac{\ddot{R}_0}{H^2 R_0} - 2\,\mu\right) z + O^2, \tag{18}$$

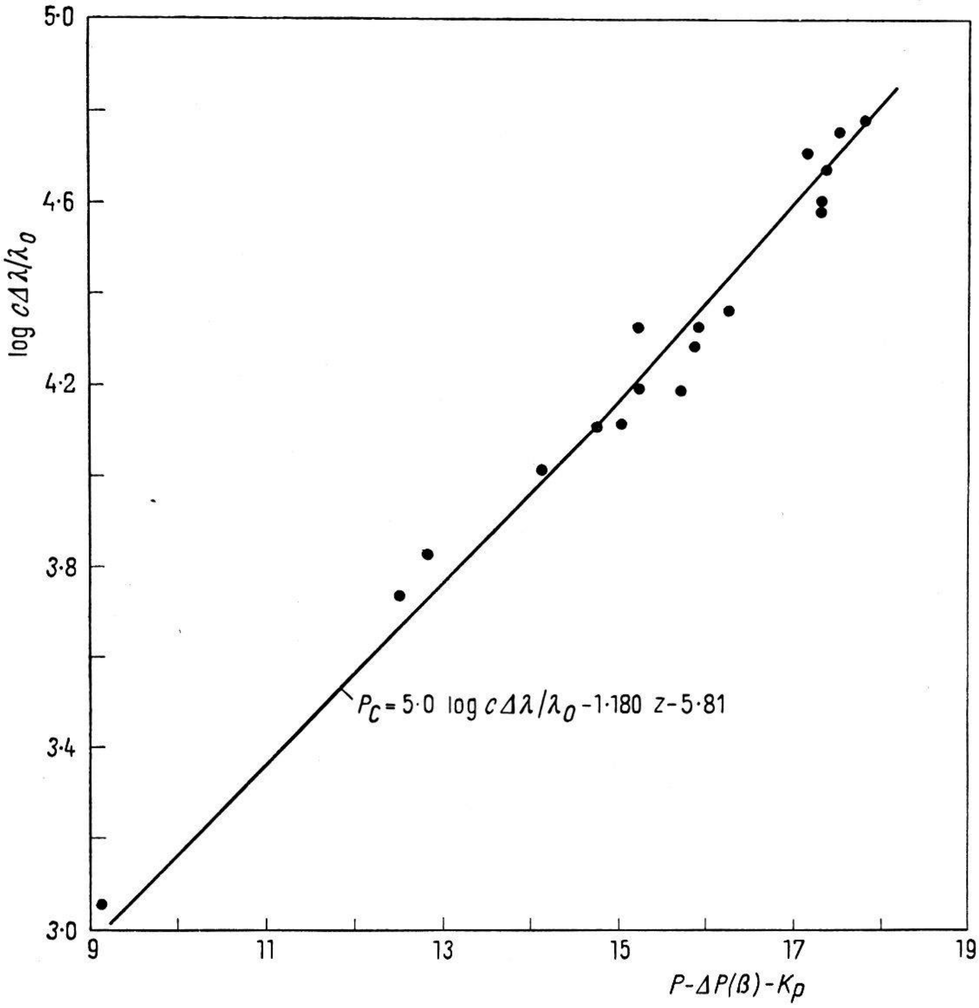

Fig. 2

where M_0 is the absolute magnitude of the nebula at the time t_0 of observation, and the constant μ is the term 0.46 $\dot{M}_0/H$ which allows for a possible change in the absolute magnitude of the nebula since the time when the light was emitted [7]. The terms on the first line are those which would give a linear 'velocity-distance' relationship of the form (15), if we simply define r in terms of m by the usual astronomical practice. If the light travels through an internebular absorbing material, the effect on the apparent magnitude can be taken into account by introducing a suitable negative term into μ. These corrections are due to effects arising outside the atmosphere; to them must be added others due to differential absorption with wave-length of light in traversing the atmosphere and the telescope, into which we will not enter in detail.

The observational material on redshifts from distant nebulae has recently been pulled together in a comprehensive survey — the first in twenty years—by HUMASON, MAYALL and SANDAGE at the Mount Wilson-Palomar and Lick Observatories, shortly to be published in the Astronomical Journal. I am deeply indebted to these authors for communi-

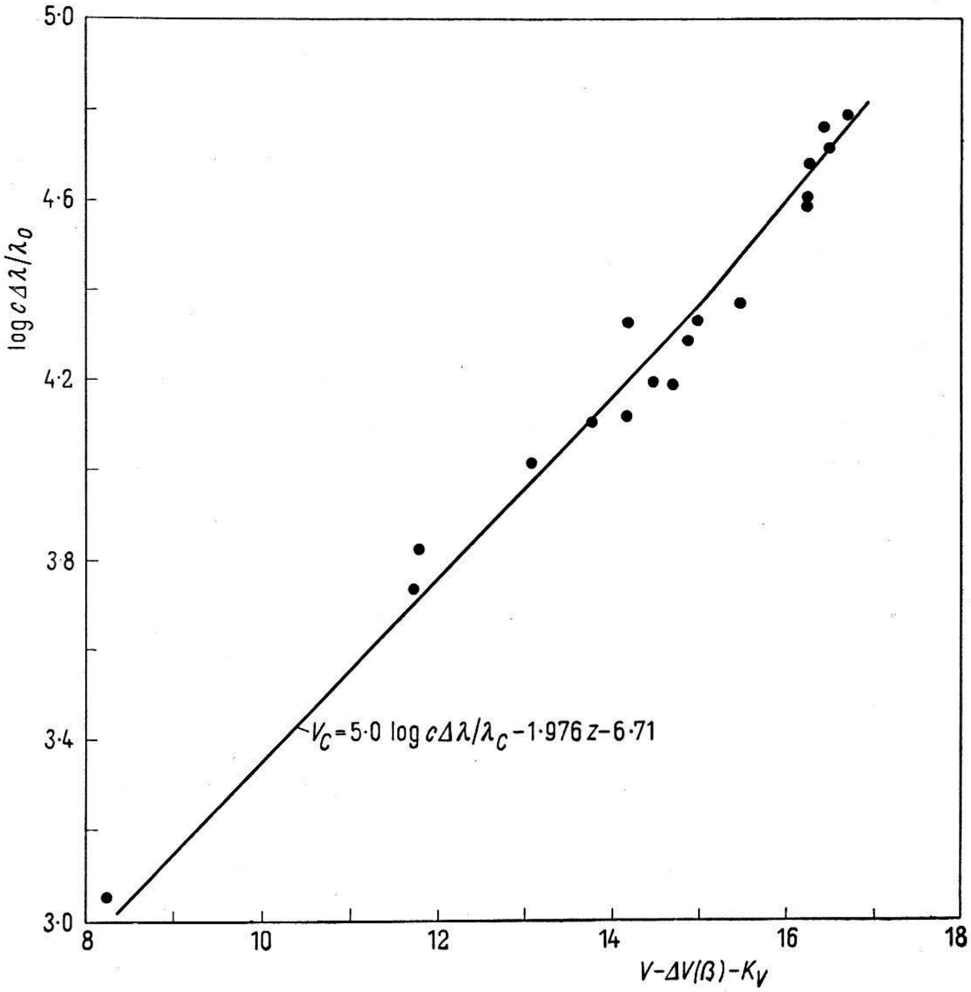

Fig. 3

cating their results to me, and for permitting me to present them to this conference. The most significant of their results for the cosmological problem is the relation they find between redshift and apparent magnitude of the brighter members of 18 clusters. The redshifts observed range up to 0.2 and the apparent magnitudes up to 18, implying velocities up to $^1/_5$ that of light and distances of over a billion light-years. Their results are given in the accompanying Figs. 2 and 3, in which the logarithm of *cz* is plotted against the photographic (Fig. 2) and the photovisual (Fig. 3) apparent magnitude, corrected in accordance with current practice for absorption within our own galaxy and for atmospheric and instrumental effects. Note that a linear velocity-distance relationship would be represented by a line parallel to the diagonal of the coordinate frame in these charts; it is apparent that the best fit of the theoretical form (18) is in each case given by a line of greater slope than the diagonal, resulting in a negative coefficient for the term linear in z.

Not included in this representation of the observational material are a number of effects which require some discussion, as they affect the coefficient of the term linear in z, and must be evaluated before we can apply the empirical results to a determination of the kinematical parameter $\ddot{R}_0/H^2 R_0$ appearing in the theoretical formula (18). The first of these is the so-called aperture effect, which arises from the fact that a relatively smaller extent of the more distant nebulae is measured than of the nearer ones. This correction slues each line about in a counterclockwise sense by about 0·2 m at its extremity, thus further increasing the non-linearity of the velocity-distance relationship. Next an estimate of the rate of change $\dot{M}$ of the absolute magnitude of an average nebula is required, in order to determine the constant appearing in the theoretical relation. SANDAGE estimates, by appeal to the theory of stellar evolution for systems of population II, that $\dot{M}$ is of the order 0·3 m per billion years. Yet another correction is required to allow for the influence of the STEBBINS-WHITFORD effect, the observed greater color index in light from the more distant nebulae. Although a definitive treatment of this must await the results of WHITFORD'S current studies, the results so far obtained lead SANDAGE to conclude that this effect just cancels the contribution due to $\dot{M}$ for the photographic case, and has no influence on the photovisual results. Finally, there is the influence of a possible uniform absorbing medium; lacking an independent estimate of its amount, we can only say that its contribution to μ, if any, would be negative.

SANDAGE'S conclusions, on applying these various corrections to the observations, is that from the photographic data the parameter $\ddot{R}_0/H^2 R_0$ cannot exceed —3.0, and that from the photovisual it cannot exceed —2·2, with a probable error due to the curve-fitting of the order of $\pm$ 0·8. Taking the mean of these two results, we may tentatively say that

$$\frac{\ddot{R}_0}{H^2 R_0} = -2{\cdot}6 \pm 0{\cdot}8\,. \tag{19}$$

I have here treated in some detail only the velocity-distance relation, but there are others which merit attention. One of these is the number count exploited by HUBBLE in his work during the 1930s, which considers the number $N(m)$ of nebulae observed out to given apparent magnitudes m. Into this count the parameter $\ddot{R}_0/H^2 R_0$ enters in a second order term, together with the curvature k/R_0^2 of space and terms depending on $\ddot{M}$ as well as on μ. Because of this complication, and because of the extreme sensitivity of the relation to density fluctuations, there seems little hope at present of getting more than a rough numerical check in this order. More promising is the possibility that we can get an empirical value of the parameter μ from the first order term, into which it enters in the coefficient $\mu - 1$ of z.

In the early days SLIPHER and others sought a measure of distance in the apparent diameter of a typical nebula. This method was supplanted later by HUBBLE'S luminosity criteria, which characterize the present-day approach to the problem of determining the distance scale, and which in the hands of BAADE and of SANDAGE has resulted in the value of H used above. But BAUM has recently revived the possibility of obtaining cosmological parameters from the photometric measurement of diameters of nebulae, or even more promising of homologous clusters of nebulae [7]. From these we may be able to obtain an independent estimate of the distance scale, and therefore of the value of HUBBLE'S constant.

Part III: The Physical Model

There remains the problem of choosing a specific model for the universe, one which will represent its past and its future as well as its present state. From the observations alone we can at most hope to get the present value of some of the kinematical parameters, such as the redshift constant H, the specific acceleration $\ddot{R}_0/R_0$ and the present value of the spatial curvature k/R_0^2. But what is required for a complete model is the full course of $R(t)$ in time, as well as the sign k of the curvature of space; this we can only get on augmenting the observational data by the imposition of physical law.

The greatest difficulty which has beset the finding of a suitable physical model in which the redshift is interpreted as DOPPLER effect rests on the fact that the earlier distance scales led to a HUBBLE constant so large that the resulting short time scale was in contradiction with the age inferred from other data. Thus HUBBLE'S own value for H around 530 km/sec per megaparsec leads, without *ad hoc* assumptions, to an upper limit of 1·8 billion years for the age of the universe, whereas there is ample evidence that the earth itself must be older than that, and that the solar system and the galaxy must be at least twice as old.

One way out of the difficulty which has appealed to some is to assume that the nebular universe is on the whole static, and that some hitherto unknown effect causes a degradation of light in travelling great distances, simulating the DOPPLER effect. The simplest assumption would be that the action responsible followed the same law as absorption, resulting in a redshift distance relation of the form

$$\frac{(\lambda + \Delta\lambda)}{\lambda} = e^{r/a}, \quad or \quad z = e^{r/a} - 1 .$$

As remarked by WHITROW [8], the expansion

$$z = \frac{r}{a} + \frac{1}{2}\left(\frac{r}{a}\right)^2 + O^3$$

can then be checked against the observations to test the validity of this type of hypothesis. Expressed in the form (18), this expansion becomes

$$m = \text{const.} + 5 \log z - 1{\cdot}086\, z + O^2 ;$$

the coefficient $-1{\cdot}086$ of z is then to be compared with the values $-2{\cdot}2$, $-1{\cdot}6$, obtained from the linear term in the empirical results before discussed on applying the indicated corrections. Considering the uncertainties in the reduction, it is seen that such an *ad hoc* explanation of the redshift cannot be rejected on the basis of present observations.

Another attempt of interest to avoid the short time scale is to assume, with HOYLE and with BONDI and GOLD, that the universe is in a steady state, and that the loss of matter in any fixed volume due to the expansion is compensated by the continuous creation of matter uniformly throughout the universe. A dynamical model of this kind must be based on the case $k = 0$, $\xi(t) = e^{t/b}$, equation (9S), where the time b is to be identified with the inverse of the HUBBLE constant. Here the quantity $\ddot{R}/H^2R$ assumes the constant value $+1$; this model is therefore inconsistent with the value $-2{\cdot}6$ of this parameter indicated by the observations. It is also at variance with the STEBBINS-WHITFORD effect, as in a steady state model there should be no systematic variation of nebular characteristics with distance. But we may look forward to hearing more concerning this theory, as well as that which JORDAN and his colleagues have been developing, in the course of this Conference.

With the longer time scale indicated by the work of BAADE and of SANDAGE, the distress is not so acute. The resulting value of $1/H$ of $5{\cdot}4$ billion years, obtained by extrapolating the present rate of expansion backward in time, appears to be of the right order of magnitude. True, gravitation, the only relevant universal force of which we have independent knowledge, tends to reduce this figure by an amount depending on the mean density of matter, for the retarding effect of the attraction will require greater nebular velocities in the past, and therefore allow a shorter time in which to reach the present state.

In order to examine this situation, I propose now to consider the models obtained on imposing the field equations of EINSTEIN's general relativity theory of gravitation – but briefly, as we are here on ground which has been trod quite thoroughly in the past. As stated at the outset, the only parameter which I shall take as given is the HUBBLE constant characterising the leading term of the velocity-distance relationship, and I shall provisionally retain the debatable cosmological constant Λ to allow a greater choice of models. Assuming, as seems justified by what we know concerning the material and energetic content of our neighbourhood of the universe, that the pressure due to radiation and to the kinetic effects of matter are negligible at the present epoch, EINSTEIN's field equations reduce for these models to the single first-order differential equation

$$8\,\pi\,G\,\varrho_0\,\xi_0^3 = -\Lambda\xi^3 + 3\xi\,(\dot{\xi}^2 + k)\,, \tag{20}$$

where G is the Newtonian constant of gravitation and ϱ_0 is the present mean density of matter. The present 'age of the universe', for those models which state out in the singular state $\xi = 0$ at time $t = 0$, is then given by the integral

$$t_0 = \int_0^1 \frac{x^{1/2}\,dx}{\left[\frac{8\,\pi\,G}{3}\,\varrho_0\,(1-x) + H^2 x^3 - \frac{k}{\xi_0^2}\,x\,(1-x^2)\right]^{1/2}} \tag{21}$$

from which Λ has been eliminated with the aid of the equation obtained from (20) for the present epoch t_0. In principle the integrand should be modified in the earlier stages, during which the radiation is of relatively greater importance, but the effect of this on t_0 can be neglected for the present purposes without substantial error.

The general nature of the dependence of the physical model on the two physical parameters ϱ_0 and t_0 is given graphically in Figure 4. Models for which $\Lambda = 0$ are represented by points on the dashed curve, which asymptotically approaches the value $t_0 = 1/H$ as the density ϱ_0 decreases. Points above this curve represent models in which $\Lambda > 0$; for these the cosmological constant acts like a repulsive force varying directly with the distance parameter and tending to counteract the gravitational deceleration. Of particular interest is the point

$$\varrho_0 = 6{\cdot}2 \times 10^{-29}\,\frac{\mathrm{g}}{\mathrm{cm}^3}\,, \qquad t_0 = 3{\cdot}6 \times 10^9\,\mathrm{yr}\,, \tag{22}$$

representing an EINSTEIN-DE SITTER universe in which both Λ and the curvature of space vanish. The signs of Λ and of the curvature, and the nature of the solution – whether oscillating or monotonically expanding – corresponding to a given pair of values ϱ_0, t_0 can be read directly off the diagram.

So far as our present knowledge of density and time scale go, it would seem possible to choose a physical model of the idealized universe in which the disreputable $\Lambda = 0$, although in order to get a large enough t_0 for the evolutionary processes we would have to keep to fairly small values of the density ϱ_0. But now comes the rub: accepting the value of the second-order term in the velocity-distance relationship indicated by the recent

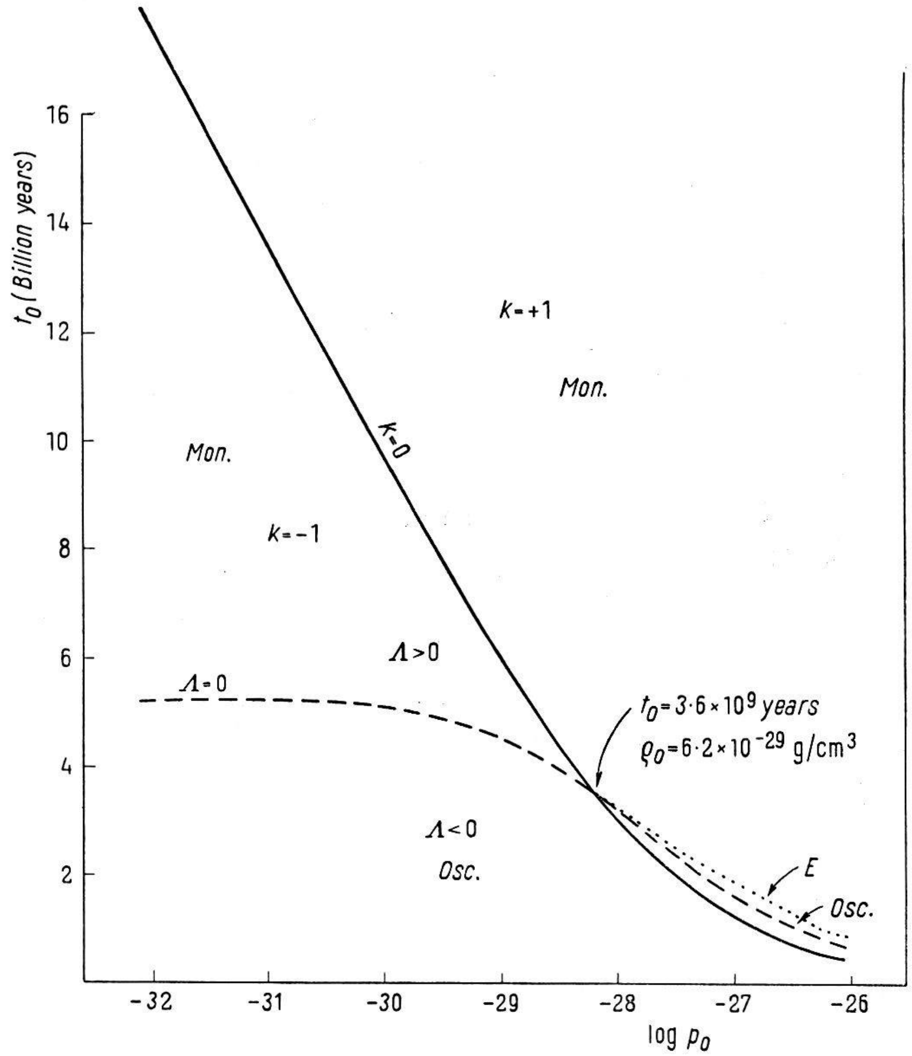

Fig. 4

survey, we are lead to such high values of ϱ_0 that we are forced to reintroduce $\Lambda > 0$ in order to save the time scale, and this in itself drives the density still higher. To show this, we go to the second-order equation obtained by differentiating (20), eliminate k between the two, and obtain an equation connecting ϱ_0 with $\ddot{\xi}_0/\xi_0$. On substituting in the resultant the value (19) obtained from the survey, we find that

$$\varrho_0 > \frac{3\,H^2}{4\,\pi G}\left(2{\cdot}7 + \frac{\Lambda}{3\,H^2}\right) = 3{\cdot}3 \times 10^{-28}\left(1 + \frac{\Lambda}{8{\cdot}1\,H^2}\right) \qquad (23)$$

But this means that if we put $\Lambda = 0$ then t_0 would be pushed down to around 2·5 billion years and we are forced to resurrect Λ to save the time scale.

With this brief glance at the models offered by EINSTEIN's general relativity theory of gravitation, I conclude my survey of cosmological theory. I make no special plea for any definite one of these models as best representing the physical universe in this, the crudest of all pertinent approximations, nor do I even insist that the model must be one chosen from this general relativistic set. It is enough to have shown that the class of kinematical models presented in Part I is prescribed by the very essence of the cosmological problem on imposing the maximum uniformity compatible with the large-scale observations, and to have shown in Part III that the general theory of relativity does lead to models adequate to portray the present semi-quantitative knowledge, presented in Part II, of the universe at large.

Diskussion – Discussion

V. FOCK: I should like to ask whether the formula you have used for $m - M_0$ is in agreement with that which follows from FRIEDMANN's solution, namely

$$\frac{rH}{c} = y + \frac{1}{2} y^2$$

$$\frac{\omega_0 - \omega}{\omega} = y + \frac{b}{4} y^2$$

where

$$b = \frac{8 \pi \gamma \varrho}{3 H^2} .$$

H. P. ROBERTSON: My formula is of course only an approximate one. If your 'r' is distance as inferred from apparent magnitude, then the two formulae agree to the approximation I am considering.

O. HECKMANN: The existence of GÖDEL's solution proves that there exists an 'absolute' rotation in the theory of relativity.

H. P. ROBERTSON: I am afraid that is correct. The entire material field in his solution must be judged to be in rotation. I consider it a defect in the field equations of the general theory of relativity that they allow such a solution.

O. HECKMANN: Could you please explain somewhat more fully the term μ considering a secular change in nebular luminosity. Do you know how SANDAGE has computed the term?

H. P. Robertson: This was inferred by Sandage from work on the theory of stellar evolution for systems of population II, by Schwarzschild and others. Sandage's own work on the globular cluster M 3 provided his estimate that the M 3 stars are about 5 billion years old.

J. Ehlers: You started with the assumption that the manifold of events is homogenous and isotropic. Therefore, you get only those models which have a cosmic time-coördinate the lines of which are orthogonal to the three-space. In connection with Gödel's model I am interested in the question: Are there arguments by which it is possible to exclude such models with an intrinsic rotation in which it is impossible to have such a time-coördinate?

H. P. Robertson: I am not aware of any argument which could enable one to exclude such models *a priori*. They do not appear among the models I discussed because I imposed both homogeneity and isotropy; Gödel's solution is homogeneous but not isotropic.

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