# The T(d,n)He^4 reaction at low energies

# The T($d$, $n$)$He^4$ Reaction at Low Energies

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*Abstract.* Recent measurement of the differential cross-sections with polarized deuterons have shown some deviation from the $s$-wave-resonance description of the reaction above 300 keV deuteron energy. An investigation has been made into the contributions coming from the interference of broad, unspecified, overlapping states with the resonance $3/2+$ state, for $E_d =$ 500 keV. The efficiency parameters $P^d_{33}$ and $P^d_{11} - P^d_{22}$ are compared with the observed values at a nearby energy $E_d =$ 580 keV.

## 1. Introduction

The usefulness of the reaction $d + t \rightarrow \alpha + n$ is well known. This reaction provides us not only with a source of $\sim$ 14 MeV neutrons but also with a polarimeter suitable for low energy deuterons [1, 2][1]). The resonance at $E_d =$ 107 KeV has been explained in terms of the $s$-wave formation of a $3/2 +$ state in the $He^5$ system [4]. This is an isolated state in so far that no higher resonance state has yet been observed in $He^5$.

Differential cross-sections have been measured by Bame and Perry [3] for deuteron energies from 0.5 MeV upwards. At such energies the departure from the $s$-wave resonance is clearly indicated by the presence of the terms proportional to $P_1(\cos\theta)$. The natural extension of the formalism to $d$-waves explains the departure from the isotropy; but again this is symmetric about 90°.

Recently P. Huber et al. [5] have measured the ratios of the differential cross-sections with polarized and unpolarized deuterons at energies 0.36, 0.50, and 0.58 MeV. These measurements also clearly indicate that above 0.3 MeV deuteron energy the reaction cannot be explained in terms of a pure $s$-wave resonance.

Goldfarb and Huq [6] have suggested a model for the reaction mechanism at low energies. They have added to the resonance contributions, a stripping amplitude, which has been treated using the D.W.B.A. Such a model of the reaction successfully explains the angular distributions at $E_d = 0.5$ MeV with the deuterons unpolarized and polarized with $P_{33}$ only. However, the angular distribution in which both $P_{33}$ and $P_{11} - P_{22}$ come in differs from the calculations and in fact, the resonance contributions alone give a better fit. This lack of agreement perhaps points to the inefficiency of this model to detect correctly the polarization of the type of $P_{11} - P_{22}$.

An alternative model for the reaction will be to consider the deuteron $d$-wave component in the $3/2 +$ resonance state together with background contributions coming from the overlapping $p$-states. Since the observed terms proportional to $P_1(\cos\theta)$ and $P_2(\cos\theta)$ are only 2–4% of the $P_0(\cos\theta)$ term, these background amplitudes will be sufficiently small to be neglected when they appear by themselves and will be significant only when they interfere with the prominent $s$-wave resonance

[1]) Numbers in brackets refer to References, page 512.

amplitude. Thus the interference of the $p$-states with the resonance explains the $P_1(\cos\theta)$ term and that of the $d$-waves with the $s$-waves explains the $P_2(\cos\theta)$ term. An investigation has been made in this line keeping the background states unspecified.

## 2. Cross-Section and Polarization Formulae

We keep in mind a reaction in which $a$ and $c$ are the spins of the target and the residual nuclei, with magnetic quantum numbers $\alpha$ and $\gamma$, respectively. Coupling $a$ to the spin $s_1$ of the projectile, channel spin. $S_1$ and coupling $c$ to the spin $s_2$ of the outgoing particle, channel spin $S_2$ are obtained. $S_1$ coupled to $l_1$ and $S_2$ coupled to $l_2$ yield the angular momentum $b$ of the intermediate state.

If the projectile is polarized and the outgoing radiation is detected with a detector not sensitive to polarization, the angular distribution is given by [7]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{pol}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{unp}} + \left(\frac{d\sigma}{d\Omega}\right)_1 .$$

Here the first term represents the ordinary angular distribution and is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unp}} = \frac{\lambda\!\!\!^{-2}}{4\,\hat{a}^2\,\hat{s}_1^2} \sum A_k(l_1\,b\,l_1'\,b';\,S_1)\,A_k(l_2\,b\,l_2'\,b';\,S_2)\,\mathrm{Re}(R\,R'^*)\,P_k(\cos\theta)\,.$$

The summation is over $k\,l_1\,l_1'\,l_2\,l_2'\,S_1\,S_2\,b$ and $b'$; $\theta$ is the angle between the outgoing and incoming beams, $\lambda\!\!\!^{-}$ is the reduced wavelength of the projectile in the centre-of-mass system, and

$$A_k(l\,b\,l'\,b';\,S) = (-)^{S-b'}\,\hat{l}\,\hat{l}'\,\hat{b}\,\hat{b}'(l\,0,\,l'\,0\mid k\,0)\,W(b\,b'\,l\,l';\,k\,S)\,.$$

Here $R$ represents the matrix element

$$\langle (c\,s_2)\,S_2,\,l_2,\,b \mid R \mid (a\,s_1)\,S_1,\,l_1,\,b\rangle$$

and $\hat{\varkappa}$ stands for $\sqrt{2\,x+1}$.

The second term which arises due to the incoming polarization, is given by

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_1 = {} & \frac{\lambda\!\!\!^{-2}}{4\,\hat{a}^2}\sum \hat{b}\,\hat{b}'\,\hat{l}_1\,\hat{l}_1'\,\hat{S}_1\,\hat{S}_1'\,A_k(l_2\,b\,l_2'\,b';\,S_2)\\ & \times \sum_{k_{1s}(\neq 0),\,k_1} \hat{k}_{1s}\,\hat{k}_1(l_1\,0,\,l_1'\,0\mid k_1\,0)\,W(s_1\,s_1\,S_1\,S_1';\,k_{1s}\,a)\\ & \times \begin{Bmatrix} S_1 & l_1 & b\\ S_1' & l_1' & b'\\ k_{1s} & k_1 & k\end{Bmatrix}(-)^{a+k_{1s}-s_1-S_1'+l_1'} \times B_1(k_{1s}\,k_1\,k;\,R\,R'^*)\,,\end{aligned}$$

where

$$\begin{aligned}B_1(k_{1s}\,k_1\,k;\,R\,R'^*) = {} & \varrho_{k_{1s}0}(s_1)\,(k_{1s}\,0,\,k_1\,0\mid k\,0)\,P_k(\cos\theta)\,\mathrm{Re}(R\,R'^*)\\ & + 2\sum_{\varkappa>0}(-)^{\varkappa}\sqrt{\frac{(k-\varkappa)\,!}{(k+\varkappa)\,!}}\,(k_{1s}\,\varkappa,\,k_1\,0\mid k\,\varkappa)\,P_k^{\varkappa}(\cos\theta)\\ & \times \begin{cases}\mathrm{Re}(\varrho_{k_{1s}\varkappa}(s_1)\,e^{i\varkappa\chi})\,\mathrm{Re}(R\,R'^*)\ \dots\ \text{even } k_{1s}\\ -\,\mathrm{Im}(\varrho_{k_{1s}\varkappa}(s_1)\,e^{i\varkappa\chi})\,\mathrm{Im}(R\,R'^*)\ \dots\ \text{odd } k_{1s}\,.\end{cases}\end{aligned}$$

The summation is over $l_1\,l_1'\,l_2\,l_2'\,S_1\,S_1'\,S_2\,b\,b'$ and $k$; $\chi$ is the angle between the normal to the reaction plane, $\boldsymbol{n} = \boldsymbol{k}_1 \times \boldsymbol{k}_2$, and the $y$-axis and $\varrho_{k_{1s}\varkappa}(s_1)$ is the spin tensor representing the incoming polarization.

In the case of deuteron, the parameterization of the polarization is as follows [8]:

$$\varrho_{00}(1) = \frac{1}{\sqrt{3}} \qquad \varrho_{20}(1) = \frac{1}{\sqrt{6}} P_{33}$$

$$\varrho_{10}(1) = \frac{1}{\sqrt{2}} P_3 \qquad \varrho_{2\pm1}(1) = \mp (P_{13} \mp i\, P_{23})$$

$$\varrho_{1\pm1}(1) = \mp \frac{1}{2} (P_1 \mp i\, P_2) \qquad \varrho_{2\pm2}(1) = \frac{1}{6} (P_{11} - P_{22}) \mp \frac{i}{2} P_{12}\,.$$

When the $y$-axis coincides with the normal $n$, the expression for the angular distribution shows that only $P_{33}$, $P_{13}$, $P_{11} - P_{22}$, and $P_2$ arise, and the form of the angular distribution is then given by [7]

$$W(\theta) = W_0 \left[1 + \frac{3}{2} P_2 P_2^d(\theta) + \frac{1}{2} P_{33} P_{33}^d(\theta) + \frac{2}{3} P_{13} P_{13}^d(\theta) + \frac{1}{6} (P_{11} - P_{22}) (P_{11}^d - P_{22}^d)(\theta)\right]$$

where $W_0(\theta)$ characterizes the unpolarized angular distribution and $P_{33}^d$, $P_{13}^d$, $P_{11}^d - P_{22}^d$, and $P_2^d$ are parameters, measuring the efficiencies of detection of the corresponding polarization parameters.

## 3. Results

Altogether seven matrix elements have been considered, one for the $S$-wave, four for the $p$-wave, and two for the $d$-wave of the deuterons. These are represented in Table 1.

Table 1

Reaction Matrix Elements

| $S_2$ | $l_2$ | $b$ | $S_1$ | $l_1$ | Symbol |
|---|---|---|---|---|---|
| 1/2 | 2 | 3/2 | 3/2 | 0 | $\alpha_0$ |
| 1/2 | 1 | 1/2 | 1/2 | 1 | $\alpha_1$ |
| 1/2 | 1 | 1/2 | 3/2 | 1 | $\alpha_2$ |
| 1/2 | 1 | 3/2 | 1/2 | 1 | $\alpha_3$ |
| 1/2 | 1 | 3/2 | 3/2 | 1 | $\alpha_4$ |
| 1/2 | 2 | 3/2 | 3/2 | 2 | $\alpha_5$ |
| 1/2 | 2 | 3/2 | 1/2 | 2 | $\alpha_6$ |

It should be noted that the $f$-wave in the exist channel has been ignored on the basis of penetrability considerations.

For convenience we introduce the dimensionless factors $c_i$ defined by

$$c_i \equiv \frac{\mathrm{Re}\,(\alpha_0\, \alpha_i^*)}{|\,\alpha_0\,|^2}, \qquad i = 1, \ldots, 6\,.$$

Terms proportional to $\mathrm{Re}(\alpha_i\, \alpha_j^*)$ are neglected on the basis of arguments in the introduction.

The angular distributions are then given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{unp}} = \frac{\lambda\!\!\!^{-2}\,|\,\alpha_0\,|^2}{6} [A_0 + A_1 P_1(\cos\theta) + A_2 P_2(\cos\theta)]\,,$$

$$\left(\frac{d\sigma}{d\Omega}\right)_1 = \frac{\lambda\!\!\!^{-2}\,|\,\alpha_0\,|^2}{6} \left[\frac{1}{2} (A_3 + A_4 P_1(\cos\theta) + A_5 P_2(\cos\theta) + A_6 P_3(\cos\theta)) P_{33} + \frac{2}{3} (A_7 P_1^1(\cos\theta) + A_8 P_2^1(\cos\theta) + A_9 P_3^1(\cos\theta)) P_{13} + \frac{1}{6} (A_{10} P_2^2(\cos\theta) + A_{11} P_3^2(\cos\theta)) (P_{11} - P_{22})\right],$$

considering deuteron polarization of the tensor type only. The vector polarization is not considered for reasons given later. The coefficients $A_i$ are given by

$$A_0 = 1 \qquad A_6 = -\frac{18}{5} C_3 - \frac{18}{5\sqrt{5}} C_4$$

$$A_1 = \frac{2}{\sqrt{5}} C_4 - C_2 \qquad A_7 = \frac{3}{4} A_4$$

$$A_2 = -2\, C_5 \qquad A_8 = -\frac{1}{2} - \frac{1}{2} C_6$$

$$A_3 = 4\, C_5 - 2\, C_6 \qquad A_9 = \frac{1}{3} A_6$$

$$A_4 = -2\, C_1 + \sqrt{2}\, C_2 - \frac{2}{5} C_3 - \frac{8}{5\sqrt{5}} C_4 \qquad A_{10} = -\frac{1}{2} + C_6$$

$$A_5 = -1 - 2\, C_6 \qquad A_{11} = \frac{1}{6} A_6 .$$

The experimental results of HUBER et al. have been measured within a constant. Choosing the constant as 6.173 the angular distribution at $E_d \simeq 490$ keV with deuteron polarization specified by $P_{33} = -0.28$, $P_{11} - P_{22} = P_{13} = 0$, can be written in the normalized form

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{pol}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{unp}} = 1 - 0.012\, P_1(\cos\theta) + 0.117\, P_2(\cos\theta) - 0.006\, P_3(\cos\theta) .$$

Using the relation

$$P_{J_1}(\cos\theta)\, P_{J_2}(\cos\theta) = \sum_J (J_1\, 0,\, J_2\, 0 \mid J\, 0)^2\, P_J(\cos\theta) ,$$

the angular distribution can be rewritten as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{pol}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{unp}} + \left(\frac{d\sigma}{d\Omega}\right)_1$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unp}} = 51.06 + 1.79\, P_1(\cos\theta) - 0.86\, P_2(\cos\theta)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_1 = -1.127 - 0.512\, P_1(\cos\theta) + 5.809\, P_2(\cos\theta) - 0.168\, P_3(\cos\theta) .$$

All cross-sections are in the unit of milibarn per steradian. The unpolarized angular distribution is taken to be that of BAME and PERRY [3] at $E_d = 500$ keV.

CONNER, BONNER, and SMITH analysed the resonance in terms of the single-level Breit-Winger formula and obtained the following parameters for $\alpha_0$: $E_\lambda = -464$ keV, $\gamma_d^2 = 2000$ keV, $\gamma_n^2 = 56$ keV. With the channel radii both taken as $5.0 \times 10^{-13}$ cm. The remaining six parameters are chosen to be consistent with $(d\sigma/d\Omega)_{\text{unp}}$ and $(d\sigma/d\Omega)_1$, thereby yielding the following values:

$$c_1 = -0.04,\ c_2 = -0.04,\ c_3 = -0.01,\ c_4 = -0.01,\ c_5 = 0.01,\ c_6 = -0.06 .$$

The resultant curve is shown in Figure 1(a) together with the experimental points.

Comparison has also been made with measurements at $E_d = 580$ keV. In the notation of Reference [6], when $\theta = \pi/2$ and $\varphi = 0$ and $\pi$, the polarization is represented by $P_{33} = 0.14$, $P_{11} - P_{22} = -0.42$, and $P_{13} = 0$. The predicted curve corresponding to the above set of parameters together with the experimental results at $E_d = 580$ keV are shown in Figure 1(b). From the comparison it can be seen that the fit is reasonably good.

We have not attempted to find $P_2^d$ in view of the fact that this is very small, being of the order of only 2% of the maximum possible value, as can be seen from Figure 11 of Reference [5]. Considering the fact that there are considerable experimental errors, it is difficult to make any definite conclusion about $P_2^d$. There are measurements corresponding to $\theta = \pi/2$, $\varphi = \pi/2$, and $\theta = \pi/2$, $\varphi = 3\pi/2$. In the first case the polarizations parameters are $P_{33} = 0.14$, $P_{11} - P_{22} = 0.42$, $P_{13} = 0$, $P_2 = 0.28$ and in the second case they are $P_{33} = 0.14$, $P_{11} - P_{22} = 0.42$, $P_{13} = 0$, $P_2 = -0.28$. Since we are not considering $P_2^d$, the two curves become identical. This has been plotted in Figure 1(c) together with the experimental results. The 'crosses' correspond to the first experiment and the 'circles' to the second experiment. The nature of agreement of the theoretical prediction with both the experiments clearly points to the fact that the effect of the vector polarization $P_2$ is negligible.

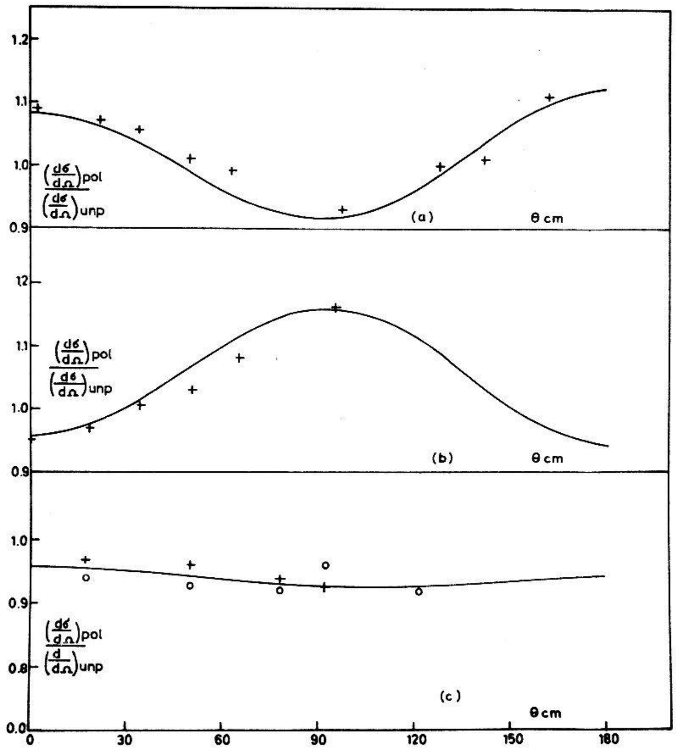


Fig. 1

Polarized angular distributions for the $d+t$ reaction at $E_d = 500$ keV, associated with deuteron polarization, (a) $P_{33} = -0.28$, (b) $P_{33} = +0.14$ and $P_{11} - P_{22} = -0.42$, (c) $P_{33} = +0.14$ and $P_{11} - P_{22} = +0.42$.

Finally we have plotted $P_{33}^d(\theta)$ and $P_{11}^d(\theta) - P_{22}^d(\theta)$ in Figure 2(a, b). The experimental points are those corresponding to $E_d = 580$ keV. Curves corresponding to the *s*-wave resonance alone are also shown. The comparison shows that some improvements are obtained in both $P_{33}^d(\theta)$ and $(P_{11}^d - P_{22}^d)(\theta)$. In Figure 2(c) we have also plotted a prediction for $P_{13}^d(\theta)$. In principle the effects of $P_{13}$ can be measured by letting $\theta = \pi/4$ and $\varphi = 0$ and $\pi$.

Summarising the results, it may be concluded that at $E_d \simeq 0.5$ MeV the 3/2 + resonance state is not the only state through which the reaction $d + t \to \alpha + n$ takes place, there are contributions from overlapping states. While the resonance 3/2 +

state is formed mostly of the $s$-waves of the deuterons, there are some $d$-wave components as well. Although the contributions from the matrix elements corresponding to the $P$- and $d$-states by themselves may be negligible, they become significant when they interfere with the $s$-wave resonance matrix element.

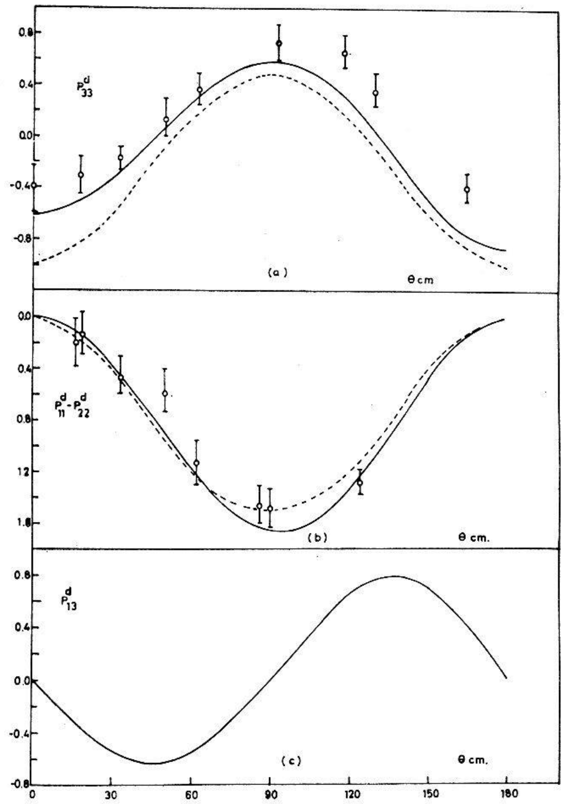


Fig. 2

The polarization efficiency parameters, (a) $P^d_{33}(\theta)$, (b) $P^d_{11}(\theta) - P^d_{22}(\theta)$, (c) $P^d_{13}(\theta)$, for the reaction $d+t$ at $E_d = 500$ keV. The dashed curve corresponds to $s$-wave resonance only.

The calculations, including the contributions from the interference terms, have a reasonably good agreement with the experimental measurements of $P^d_{33}(\theta)$ and $P^d_{11}(\theta) - P^d_{22}(\theta)$. Although the calculations have been done at an energy of 500 keV, and the experimental results correspond to 580 keV, effects due to this much variation in energy are not expected to be great. It will be of interest to examine the validity of the assumptions and conclusions by performing some experiments on the detection of $P_{13}$ and comparing the values of $P^d_{13}(\theta)$ with the prediction.

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