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On Properties of Unstable Particles

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Abstract. The condition is postulated which determines possible decay laws of unstable particles. These decay laws are interpreted in terms of single and multiple poles on the second sheet.

1. It is well known that the exponential decay law is not the only possible decay law of unstable particles. Various models of mixing give decay laws in the form of a superposition of exponentials [1]. Further, possible poles of higher order in amplitudes are expected to produce decay laws of the form: an exponential times polynomials in time [2]. One could ask whether also other possibilities do exist.

In order to answer this question and to find all possible decay laws we have to postulate some general property of unstable particles allowing to distinguish them from other states and their decay from other time evolutions. The present approach to this subject is based on the natural assumption that any decay is always accompanied by the production of outgoing decay products which can no longer reproduce the unstable particles. This assumption, distinguishing the decay from the mutual transitions of unstable particles (like in mixing) leads to an equation which determines the possible decay laws in dependence on the number N of the mutually transiting unstable particles.

The interpretation of the decay laws thus obtained in terms of the singularities on the unphysical sheet is made by means of the resonance approximation of the scattering amplitudes of the decay products. Then any solution for finite N turns out to produce a set of single or multiple poles. Further, the unitarity condition relates the parameters describing the decay laws to the decay vertices of the unstable particles. For illustration, a particular case of the pole of second order is considered in some detail.

2. Let a set of N mutually transiting unstable particles U_α be described by some set of orthogonal states $|U_\alpha\rangle$: These states are characterized by zero velocity so that their time evolution does not include spatial montions of the unstable particles[2]. This time evolution can then be written in the form

$$e^{-iHt}\,|U_\alpha\rangle = \sum_{\beta=1}^{N} a_{\alpha\beta}(t)\,|U_\beta\rangle + |\psi_t^\alpha\rangle\,,\ \alpha = 1, \dots, N;$$

$$a_{\alpha\beta}(0) = \delta_{\alpha\beta}\,,\ \langle U_\gamma|\psi_t^\alpha\rangle = 0\,, \tag{1}$$

[1]) On leave of absence from Charles University, Prague.

[2]) Such a possibility of the description of unstable particles is discussed in some detail elsewhere [3].

where the states $|\psi_t^\alpha\rangle$ describe the decay products of the particles U_α which are present at the moment $t > 0$. Our assumption is that these states fulfil at least approximately the condition

$$\langle U_\gamma | \, e^{-iHt'} \, |\psi_t^\alpha\rangle = 0 \tag{2}$$

for any α, β and $t, t' > 0$.

Intuitively, this assumption is consistent with a good localisability of the unstable particles. The decay products leave immediately after their production the interaction region which is given by the size of the unstable particles and can no longer reproduce an unstable particle.

On the other hand, if some resonance state of the decay products allowed them to remain longer in the interaction region and thus violate (2), such a state should be included into the set of the unstable particles U_α [3]). In this sense the exactness of the condition (2) depends on how many states mutually transiting with some unstable particles during their decay are included into their set.

From (1) and (2) the matrix equation

$$A(t) \, A(t') = A \, (t + t') \, , \; [A(t)]_{\alpha\beta} = a_{\alpha\beta}(t) \tag{3}$$

follows immediately. Presumably it determines possible decay laws in dependence on the number N. The solution for finite N is apparently of the form

$$A(t) = \sum_{n=0}^{\infty} \frac{1}{n!} \, B^n \, t^n \tag{4}$$

where B is an arbitrary N-dimensional matrix. We note that provided the Hamiltonian H and the states $|U_\alpha\rangle$ are T-invariant the matrices $A(t)$ and B are symmetric.

3. For the classification of the solutions (4) it is convenient to write the matrix B in the Jordan form [4]

$$B = V \, \{ \ldots , (- i \, \mu_j + F_j), \ldots \} \, V^{-1} \tag{5}$$

where the brackets {} mean a quasi-diagonal N-dimensional matrix with N_j-dimensional matrices on the diagonal. Here

$$[F_j]_{\alpha\beta} = \delta_{\alpha, \beta-1} \qquad j = 1, \ldots , r; \quad \sum_{j=1}^{r} N_j = N \, . \tag{6}$$

The matrix V is any arbitrary regular matrix and r is the number of the matrices on the diagonal. The parameters μ_j are written in the form

$$\mu_j = \sqrt{M_j^2 - i \, M_j \, \Gamma_j} = M_j - \frac{i}{2} \, \Gamma_j \, , \; 0 \leqslant \Gamma_j \ll M_j \tag{7}$$

which excludes increasing of the amplitudes $a_{\alpha\beta}$ with time.

[3]) Here, it is only a matter of convenience whether the mutually transiting unstable particles are called different particles, different states of some particle or analogously.

In the case $r = 1$ the matrix $A(t)$ can be rewritten in the form

$$A(t) = e^{-i\mu t} \sum_{n=0}^{N-1} \frac{1}{n!} R^n t^n , \quad R = V F V^{-1} , \tag{8}$$

where the summation is finite because the matrix R is nilpotent. The decay laws obtained from this solution are just of the form expected for the pole of N-th order [2]. The case $r > 1$ can be interpreted as a mixing of several above cases.

It is worth mentioning that a non-exponential decay law is allowed by equation (3) only if $N > 1$. This is easily understood on the basis of the condition (2): For a non-exponential decay, some mechanism for remembering the age is necessary but because of (3) the decay products cannot serve for this purpose. Thus only the existence of more mutually transiting particles can cause deviations from the exponential.

We note that the solution (4) are not in agreement with the constraint

$$\int_{-\infty}^{+\infty} dt \, \frac{|\ln |a_{\alpha\beta}(t)||}{1 + t^2} < \infty \tag{9}$$

which is, according to the Paley-Wiener theorem [5], a consequence of the positive definiteness of the energy [6, 7]. Thus the condition (2) cannot be exact if the number of the unstable particles N is assumed to be finite. However, the effects of the positive definiteness of energy are important only for $t \gg 1/\Gamma$ [7, 8] and the solutions with finite N can thus represent reasonable approximations of the decays.

4. The resonance effects caused by unstable particles decaying according to equation (4) can be obtained by means of the propagator approximation of the amplitudes. The corresponding propagator matrix is of the form

$$\Pi^{(+)}(s) = V \{\ldots , (s - \mu_j^2 - 2 i \mu_j F_j + F_j^2)^{-1}, \ldots\} V^{-1} . \tag{10}$$

Its poles are given by the equations

$$(s - \mu_j^2)^{N_j} = 0; \quad j = 1, \ldots , r \tag{11}$$

which, because of (7), are situated on the second sheet or eventually on the real axis. We note that a pole of the N-th order is connected at least with N unstable particles.

The formation scattering amplitudes are then assumed to be of the form

$$T_{ij}(s) = i (S_{ij} - \delta_{ij}) = \sum_{\alpha, \beta = 1}^{N} g_\alpha^{*(j)} \Pi_{\alpha\beta}^{(+)}(s) \, g_\beta^{(i)}; \quad i, j = 1, \ldots , c \tag{12}$$

where $g_\alpha^{(i)}$ is the constant decay vertex of the particle U_α into an i-th channel. The unitarity condition for these amplitudes restricts possible values of g and relates them to the parameters which determine the matrix B. With use of these vertices also the production processes can be written

$$T_i(s) = \sum_{\alpha, \beta = 1}^{N} G_\alpha \, \Pi_{\alpha\beta}^{(+)}(s) \, g_\beta^{(i)} \tag{13}$$

the constants G_α remaining without further dynamical assumption undetermined, however.

5. To illustrate the present approach we consider in some detail the case $N = 2$, $r = 1$ which corresponds to the pole of the second order in the amplitudes. This case has been considered by various authors [9] mostly in connection with the structure of the A_2-meson peak [10]. The general form of the symmetric nilpotent matrix R in equation (8) is

$$R = \varrho \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} \tag{14}$$

where the parameter ϱ can be made real by a proper choice of the basis $|U_1\rangle$, $|U_2\rangle$. On neglecting the terms Γ^2 and $\Gamma \cdot \varrho$ the propagator is then of the form

$$\Pi^{(+)}(s) = \frac{1}{(s - M^2 + i\,M\,\Gamma)^2} \times \begin{pmatrix} s - M^2 + i\,M\,(\Gamma + 2\,\varrho) & 2\,\varrho\,M \\ 2\,\varrho\,M & s - M^2 + i\,M\,(\Gamma - 2\,\varrho) \end{pmatrix}. \tag{15}$$

The unitarity condition is easily solved in the case of one-channel decay $c = 1$. One non-trivial solution is

$$g_1 = \sqrt{M\,\Gamma}\,, \quad g_2 = \pm\, i \sqrt{M\,\Gamma} \tag{16}$$

with arbitrary ϱ leading to the T-matrix of the form

$$T(s) = \frac{2\,M\,\Gamma}{s - M^2 + i\,M\,\Gamma}\,. \tag{17}$$

The disappearance of the pole of the second order is due to the fact that the states

$$|U^{\pm}\rangle = \frac{1}{\sqrt{2}}\,(|U_1\rangle \pm i\,|U_2\rangle) \tag{18}$$

decay, in consequence of equations (8) and (14), exponentially for $t \lessgtr 0$. The second order pole in the form

$$T(s) = \frac{4\,M\,\Gamma\,(s - M^2)}{(s - M^2 + i\,M\,\Gamma)^2} \tag{19}$$

is obtained in two ways

$$g_1 = 0\,, \quad g_2^*\,g_2 = 4\,M\,\Gamma\,, \quad \varrho = \frac{\Gamma}{2} \tag{20}$$

or

$$g_1^*\,g_1 = 2\,M\,\Gamma\left(1 - \frac{\Gamma}{2\,\varrho}\right), \quad g_2^*\,g_2 = 2\,M\,\Gamma\left(1 + \frac{\Gamma}{2\,\varrho}\right)$$

$$\frac{g_1}{|g_1|} = \pm\, i\,\frac{g_2}{|g_2|}\,, \quad |\varrho| > \frac{\Gamma}{2}\,. \tag{21}$$

The formula (19) is a possible explanation of the structure of the A_2-meson in its dominant $\varrho\,\pi$ decay channel [11].

In the case of more decay channels, various amplitudes are apparently possible. One set, which is an extension of (20), is given by the equations

$$\sum_{j=1}^{c} g_1^{*(j)}\, g_1^{(j)} = 2\,M\,(\Gamma - 2\,\varrho)$$

$$\sum_{j=1}^{c} g_2^{*(j)}\, g_2^{(j)} = 2\,M\,(\Gamma + 2\,\varrho)$$

$$\sum_{j=1}^{c} g_1^{*(j)}\, g_2^{(j)} = 0 \tag{22}$$

which allowe amplitudes to differ from the typical double peaked amplitude (19). Further variations of the amplitudes are possible due to the free parameters G_α in the production processes. For example, the explanation of the one-peak structure of the $K\overline{K}$ decay mode of the A_2-meson [12] has been given in this way [13].

It could be important to stress that some possible amplitudes produced by the pole of the second order are hardly distinguishable from the single pole. For example, in the two-channel case it is possible to choose, in agreement with (22),

$$g_1^{(1)} = g_1^{(2)} = 2\sqrt{\frac{2}{5}\,M\,\Gamma}$$

$$g_2^{(1)} = -\,g_2^{(2)} = \sqrt{\frac{2}{5}\,M\,\Gamma}$$

$$\varrho = -\,\frac{3}{10}\,\Gamma \tag{23}$$

which, provided $G_1 \sim G_2$, leads to the production amplitudes

$$9\,|T_2|^2 = |T_1|^2 = \frac{2}{5}\,G^2\,M\,\Gamma\,\frac{(s - M^2 - 3/5\,M\,\Gamma)^2 + (4/5\,M\,\Gamma)^2}{[(s - M^2)^2 + M^2\,\Gamma^2]^2}\,. \tag{24}$$

Their forms, identical in both channels, can easily be misinterpreted as a Breit-Wigner resonance.

Finally, we shall establish, in our approach, the second order pole as a limit of two nearby poles. It is a straightforward procedure to show that the matrix B^2 can be written in the case $N = r = 2$ in the form

$$B^2 = \begin{pmatrix} -\mu_1^2 - p\,\Delta & i\,\Delta\,\sqrt{p^2 + p} \\ i\,\Delta\,\sqrt{p^2 + p} & -\mu_2^2 + p\,\Delta \end{pmatrix},$$

$$\Delta = \mu_1^2 - \mu_2^2 \tag{25}$$

with p real. The corresponding propagator evidently becomes identical with the propagator (15) in the limit

$$\mu_1^2 \to \mu^2\,, \quad \mu_2^2 \to \mu^2\,, \quad p \to \infty \qquad p\,(\mu_1^2 - \mu_2^2) \to 2\,i\,M\,\varrho\,. \tag{26}$$

One will readily verify that this limit calculation can be performed in agreement with unitarity.

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