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Remarks on an Axiomatic Approach to Quantum Gravity[1]

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Abstract. Problems raised by an alternative quantization of gravity in an axiomatic manner are discussed and clarified. In particular, explicit arguments concerning the spin of the graviton and the stability of the geometrical light-cone are presented.

In a series of papers [1–3] a new approach to an axiomatic quantization of the gravitational field was presented. Since this alternative is quite different in concept as well as in some of its consequences from the usual approaches, it seems important to us to present in this short note some arguments in favour of the main ideas of these papers as well as some clarification concerning the problem of spin and local covariance of the gravitational field.

If one wants to keep to the spirit of general relativity, also in its quantized version, the most obvious way to do so is [1–3] to postulate that it is the quantized gravitational field which generates its own background geometry. In turn this geometry should consist of a well-defined and fixed light-cone field if any meaningful formulation of a quantum field theory is expected.

Attempts towards a plausible justification for the breaking of the fixed light-cone field appearing in most of the quantum gravity theories can be mainly divided into two directions:

1) quantization of space-time itself, and
2) oscillations of the quantized metric-field.

We shall now sum up the main arguments put forward in favour of these two directions from a critical point of view. In a later part of the present note additional arguments which support the fixed light-cone field theory will be developed.

1) The problem of quantization of space-time is rather old. It is quite clear that when passing from classical to quantum theories there is no *a priori* reason to keep the structure of space-time completely unchanged. The main reason for not changing the structure of space-time in quantum theory is that the problem of quantization of space-time itself is very difficult and not well-defined mathematically. Moreover, the success of quantum electrodynamics in giving rules for calculating phenomena based

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[1]) Part of this work was done at the Institut de Physique Théorique de l'Université de Genève.

on interactions of electrons and photons without quantizing space-time itself made people less interested in quantization of space-time.

It was claimed by some people [4] that if the underlying space-time is not itself subject to quantization, it is even not operationally defined. Indeed, according to this well-known argument, in order to localize the position of a particle in the background space-time, a high uncertainty in the momentum (and energy) of the test-particle is introduced, which can destroy the whole input structure by pair-creation.

Evidently a more sophisticated construction of position operators of physical particles, based on imprimitivity systems (and their generalizations), can solve this kind of paradox.

2) Concerning the problem of oscillation of the quantized metric field [4] two classical arguments are utilized in favour of this phenomenon. The first argument states that if the field is induced by a turbulent motion of matter it becomes a kind of *stochastic quantity*. This quantity – under the assumption of not too big fluctuations – can be considered as an average fixed metric plus small perturbative term.

Now, as one would certainly like to begin with a quantization of the gravitational field in vacuum (before taking into consideration complicated possible cosmological situations), this argument can be disregarded for our purpose.

The second argument, originally due to Pauli, is that if oscillation of the metric really takes place in nature it might have an extremely important effect on our understanding of quantum field theory. Indeed these fluctuations in the metric might smear out the singularities, and in particular it might force the famous ultra-violet singularities to disappear.

This argument is certainly very beautiful. However, one is not obliged to take it too seriously, and that because ultra-violet singularities are not the only difficulty we have in field theory (other difficulties exist which do not have an immediate connection with the gravitational field) and besides up till now nobody has really proved that cancellation of ultra-violet divergences takes place if one takes into account fluctuations of the light-cone. On the other hand, if such fluctuations are admitted, we have to pay a very high price: past and future of a given event are not absolute and can, in principle, mix up. Space-like separated events in an oscillating cone-field is also not a well-defined concept, as these events can at the same time (relatively to another cone within the oscillation domain of uncertainty) manifest a time-like separation character. Consequently, basic notions like local-commutativity, spectrality, etc. . . . become obscure, and therefore no serious attempt to formulate a quantum field theory of gravity without radical conceptual changes can be made.

This is the main philosophy which guided us in our attempt to develop a rigorous quantization scheme for the gravitational field [1, 3].

In this series of papers, one of the preliminary results was the following: the quantized gravitational field $\mathcal{G}$, being a distribution-metric, must be the product of a usual (scalar) operator-valued distribution λ times a classical metric g. This result, however, does not mean that 'at each point we get proportional operators', which one could be misled to think by using the distribution δ as test-function (and considering the distribution-metric as a function). In fact such a distribution-metric, when smeared with a test-function, can in general give rise to ten linearly independent operators; we do have room for 'ten degrees of freedom'. This can be illustrated by the following simple example. Suppose λ has point-masses at the points x_k $(k=1, \ldots, K)$: $\lambda = \sum_{k=1}^{K} \delta(x - x_k)\, A_k + \omega I$, where the A_k and the identity I are linearly independent operators, and ω is an orientation. Then, for any given test-function φ, we have (denoting by $C_{\mu v} = \int \varphi g_{\mu v} \omega$)

$$\mathcal{G}_{\mu\nu}(\varphi) = \lambda g_{\mu\nu}(\varphi) = \sum_k g_{\mu\nu}(x_k)\, \varphi(x_k)\, A_k + C_{\mu\nu} I.$$

If these operators are linearly dependent, we should have, for some $\{\lambda_{\mu\nu}\}$ not all zero,

$$\sum_{\mu\nu} \lambda_{\mu\nu}(\sum_k g_{\mu\nu}(x_k)\, \varphi(x_k)\, A_k + C_{\mu\nu} I) = 0$$

and therefore, the A_k and I being independent, $(\sum_{\mu,\nu} \lambda_{\mu\nu} g_{\mu\nu}(x_k))\, \varphi(x_k) = 0$ for all x_k.

Since we can choose test-functions φ not vanishing at enough points x_k, the latter implies that if K is large enough ($K \geqslant 10$), except for very particular metrics $g_{\mu\nu}$ (conformal to a constant one for example), all $\lambda_{\mu\nu}$ vanish. This simple example is characteristic of the general phenomenon, since the algebraic tensor product of the space of ordinary distributions by that of operators is dense in the space of operator-valued distributions. Any operator-valued distribution λ is therefore a limit of finite linear combinations $\sum \lambda_k A_k$ of products of ordinary distributions λ_k multiplied by operators A_k; if the field has enough singularities, we shall in general have place for ten operatorial degrees of freedom as limits of the $\sum \lambda_k(g_{\mu\nu}\varphi) A_k$ when smearing with a test-function φ.

Of course, like in the usual theories, this number of operatorial degrees of freedom can be reduced by imposing additional conditions.

The above-mentioned restriction (factorization of $\mathcal{G}$) is related to the geometrical definition which is given of the isotropic vectors and light-cone field. One could then be tempted to weaken these definitions and have them depend upon the states, i.e. define isotropic vectors X relative to states ψ and ψ' by $(\psi', \mathcal{G}(X \otimes X \otimes \varphi)\psi) = 0$, or only upon the test-function φ.

The former, however, would have no meaning. Indeed, should we attach to each state vector (or pair of state vectors) a cone-field, all the construction would be indexed by state vectors, including, for example, the definition of the local energy-momentum operator and its expectation values: in this case, for each state, only one matrix element of the corresponding energy-momentum operator would have a physical sense. One could have argued (in accordance with the experience we gained in the last years in quantum-field theory) that since in any case the vacuum-state Ω plays an exceptional role in field theory it would be reasonable to define the classical metric (and with it the cone-field) by the equation:

$$(\Omega, \mathcal{G}_{\mu\nu}(x)\, \Omega) = g_{\mu\nu}(x)$$

(i.e. $(\Omega, \mathcal{G}_{\mu\nu}(\varphi)\Omega) = \int g_{\mu\nu}(x)\varphi(x)\,dx$ for every test function φ). However, the usual Poincaré covariance of the tensor field $\mathcal{G}_{\mu\nu}$ and the Poincaré invariance of the vacuum would then trivially imply the *flatness* of the classical metric $g_{\mu\nu}$. We would then have a complete decoupling between the quantum field and its classical background, as well as a trivial solution to the underlying classical Einstein equations.

The latter alternative also would not be physically very sensible, and in addition would turn to be mathematically more or less equivalent to the one we chose. For a test-function is physically interpreted as a signal sent to measure the field, and, for example, the notions of separation (time-like, space-like, light-like) should not depend on such a signal.

Mathematically, let us take two test functions φ and φ' strictly positive at the point x on the manifold in which we define the cone-field. We may then, if the support of X_μ is small enough (which we can assume), write $\varphi' = \alpha\varphi$, with α a positive function, in this support.

Then $\mathscr{G}^{\mu\nu}(X_\mu X_\nu \varphi') = \mathscr{G}^{\mu\nu}(\alpha^{1/2} X_\mu \cdot \alpha^{1/2} X_\nu \varphi)$ and the isotropic vectors defined by φ' are proportional to those defined by φ. This shows that even if one insists on making the light-cone field depend on φ, it is rather implausible that this might constitute a real mathematical generalization.

As was noticed in Refs. [1–3], as well as, in a similar context, in Ref. [5], one of the most difficult axioms to satisfy by quantum theory of gravity is the axiom of (local) covariance.

In Ref. [5] an explicit suggestion is made of a possible modification of this covariance axiom. If one takes this suggestion seriously, or if one postulates a similar (but more plausible) new covariance axiom, in which the unitary representation of the Poincaré group is replaced by that of the semi-direct product of Poincaré by a gauge group, difficulties still occur connected with some strong conditions which have to be satisfied by the gauge, with the connection of the latter to the famous problem of the 'indefinite-metric Hilbert space', etc. . . .

Besides the problem of indefinite metric itself, other arguments based on the free-field case exist in favour of the utilization of non-unitary representations of the Poincaré group in the covariance axiom for the massless case [6]. Indeed, in the massive case (and for free field), a simple connection exists between $S(\Lambda)$ and $U(a,\Lambda)$ where (for flat space and, for example, Dirac field) we denote

$$U(a,\Lambda)\,\psi(\varphi)\,U(a,\Lambda)^{-1} = S(\Lambda^{-1})\,\psi(\varphi(\Lambda^{-1}(x-a)));$$

the restriction of $S(\Lambda)$ on the little group (in this case $SU(2)$) is the inducing representation of the restriction of $U(a,\Lambda)$ on the one-particle states. If we want this connection to hold also in the massless case, the corresponding $U(a,\Lambda)$ will be non-unitary.

However, the impossibility of giving the usual full meaning to the covariance axiom for gravitational field, which was proved in Ref. [3], holds, unfortunately, also for non-unitary representations of the covariance group. (It should be mentioned also that if in this case we take $O(2)$ instead of $E(2)$ for the little group, we get a unitary representation, in general reducible, on the 'massless one-particle states'.)

In Wightman quantum field theory one takes a different point of view than above. One ignores any connection which might exist between $S(\Lambda)$ and U, and just *imposes* in the Haag–Ruelle theory of asymptotic states (which is necessary in order to define the notion of particle in Wightman theory) on the one-particle states to have a given spin. One should notice that indeed, in the Wightman theory, one has to impose this definition of spin. If this is not done, simple examples can show how the definition of spin in the asymptotic states sense need not coincide with the possible definition via $S(\Lambda)$. (One can start with a covariance axiom for a tensor field having the 'normal' connection between $S(\Lambda)$ and U, contract with a suitable field of real-valued tensors, and get after contraction an 'anomalous connection' between $S(\Lambda)$ and U.) Therefore, though in our approach we cannot maintain the covariance axiom in the usual sense [1–3] (a thing which is not astonishing since the notion of spin is different for massless particles from the massive case already in flat space-time theories and causes difficulties already there), one can still impose in the corresponding Haag–Ruelle theory of asymptotic states on the one-particle states to have the usual two helicity states ± 2. It is also possible, in our framework, to get additional (possibly all) possible helicity states of the graviton, like in the scalar-tensor theory or in the Rosen theory of gravitation, simply by using the appropriate representations (possibly non-unitary and even indecomposable) of the Poincaré group. Whether to treat the *quantized* graviton as a scalar particle or as a 'spin-two, massless' particle in the asymptotic states theory sense is, and will be in the coming decades, an academic question.

We shall end our note with the following two remarks:

1) Unless stated differently (as in the last section) our remarks concern the *interpolating gravitational field* on which no *a priori* equations of motion are imposed and *not* the asymptotic gravitational fields which satisfy the Einstein equations.

2) One might argue that since Wightman's axioms in Minkowski space contain some elements of quantum measurement theory they are not the best possible framework for quantum gravity, as the gravitational field causes difficulties from the point of view of observables already on a classical level. The answer to such an objection is the following:

a) We feel that it is better to have mathematically well-defined set of axioms than something undefined and arbitrary.
b) The observable character of Wightman theory is an open question already in Minkowski space. If, for example, a real scalar Wightman field is not a generalized free field, one does not know in general if such a field is an observable in the quantum mechanical sense!
c) If by measurement theory for an interpolating field one understands some weak form of microcausality or the existence of some type of generalized local energy-momentum operators – which are indeed extremely important for the theory to have a plausible physical interpretation – then our approach meets these requirements. Other properties, like field operators being observables, are not necessary *a priori* for the interpolating field – but one would hope to get them as a consequence of the whole set-up. This problem, however, is yet unsolved for the Minkowski-space case, as mentioned above.

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