Zeitschrift: Helvetica Physica Acta
Band: 50 (1977)
Heft: 6

Artikel: Light cone sum rules for single-pion electroproduction

Autor: Mallik, S.

DOI: https://doi.org/10.5169/seals-114893

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 06.10.2024

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

Light cone sum rules for single-pion electroproduction[1])

by **S. Mallik**

Institute for Theoretical Physics, University of Berne, Sidlerstrasse 5, CH-3012 Berne, Switzerland

(25. VII. 1977)

Abstract. We derive light cone dispersion sum rules (of low energy and superconvergence types) for nucleon matrix elements of the commutator involving electromagnetic and divergence of axial vector currents. The superconvergence type sum rules in the fixed mass limit are rewritten without requiring the knowledge of Regge subtractions. The retarded scaling functions occurring in these sum rules are evaluated within the framework of quark light cone algebra of currents. Besides a general consistency check of the framework underlying the derivation, we infer, on the basis of crude evaluation of scaling functions, an upper limit of 100 MeV for the bare mass of nonstrange quarks.

1. Introduction

It is well-known that causality of current commutators can be exploited [1, 2] to write down light cone dispersion relations (LCDR) and light cone sum rules (LCSR) for their Fourier transforms. Such relations, in general, involve subtraction constants depending on the behaviour of the commutators on the light cone. The quark light cone algebra of currents [3] allows these constants or the retarded scaling functions to be expressed [4] in terms of matrix elements of quark bilocal operators. Since the bilocals are not described by free quark fields, it is, of course, not possible to evaluate them without detailed knowledge of quark interaction. Nevertheless, it is possible to extract some model independent information about them from certain sum rules satisfied by the absorptive scaling functions.

The LCDRs and LCSRs involve, in general, both timelike and spacelike values of masses of external currents, making it difficult to apply them phenomenologically. One has, therefore, to consider the limit of fixed mass in the dispersion integrals. The procedure [4] consists in subtracting out from the amplitude a Regge piece corresponding to some of the leading J-plane singularities of the fixed mass amplitude, such that the sum rule for the subtracted amplitude allows one to go to the fixed mass limit. These relations are then a useful source of information on the scaling functions from a phenomenological point of view.

The present work consists in studying sum rules for amplitudes involving the electromagnetic and the divergence of the axial vector currents. These amplitudes are related to the exclusive one pion electroproduction process. In addition to the LCSRs (to be referred as superconvergence type sum rules) we also consider low energy type sum rules obtained by combining low energy theorems [5, 6] with LCDRs.

[1]) Work supported by the Schweizerische Nationalfonds.

It is easy to realize that the Regge subtraction involved in these sum rules is an extremely delicate affair. In the language of finite energy sum rules (FESR) [7], when the Regge piece, R, is extrapolated in the low energy region, it is the deviation from average duality (the deviation is expected since we have not included all singularities in R) which gives essentially the scaling function. A tolerable accuracy at the level of non-leading singularities is indeed hard to achieve even in the best known case of Regge parametrization, namely, πN scattering, not to mention pion photoproduction where simple high energy Regge parametrization fails drastically to satisfy FESR [8].

To avoid the difficulty with Regge subtraction in the superconvergence type sum rules, we eliminate the Regge piece between the original sum rule for zero pion mass amplitude and the corresponding one for the on-shell amplitude. The sum rules in this form have the additional advantage of showing explicitly that all terms are formally of the order of m_π^2. These are then trivially satisfied in the chiral limit.

Because of the presence of divergence of the axial vector current in our amplitude, the bare mass of the nonstrange quarks, m_q, appears in the scaling functions. This is one of the parameters responsible for chiral symmetry breaking. One expects that a numerical evaluation of our sum rules would not only allow a general consistency check of the framework involved in their derivation but also a quantitative determination of m_q. But since we do not wish to commit ourselves to any specific model, we are forced to estimate the retarded scaling functions from just a few integral relations for the absorptive scaling functions. Because of this difficulty, we can only argue for an upper limit to m_q.

Although the present work assumes exact scaling as given by the quark light cone algebra of currents, it is easy to modify them if scaling is violated. We discuss this possibility in the context of asymptotically free gauge theories [9]. The singularity structure of the commutators on the light cone predicted by these theories is slightly smoother than the canonical singularity given by the light cone algebra. As a result all the light cone sum rules of this paper will remain valid in these theories with the retarded scaling functions being calculated according to such theories. From the behaviour of the moment integrals over the structure functions, it can be readily seen that the retarded scaling functions in the truely Bjorken limit is proportional to the integral over the absorptive scaling functions and hence are not expected to differ much from that given by the light cone algebra. In particular, the parameter m_q in the scaling functions may be interpreted [10] as the quark mass relevant in the non-asymptotic but approximate scaling region. In the fixed mass limit, however, the situation is more complicated.

Section 2 deals with kinematics of the process considered. We also discuss the constraint on the invariant amplitudes imposed by current conservation. In Section 3 we write the generalized Bjorken limit of our amplitude in terms of the matrix element of the quark bilocal operator, decompose the latter in terms of form factors and finally derive the scaling functions for the invariant amplitudes in terms of these form factors. In Section 4 we rederive the low energy theorems for the invariant amplitudes free from kinematic singularities. We write down the light cone sum rules (low energy and superconvergence types) in Section 5. In Section 6 we eliminate the pion pole contribution in t channel, when present, from both sides of the sum rules. This not only provides a partial check on these but also serves partly to carry out the Regge subtraction to be performed in the next section. We then go to the fixed mass limit of these sum rules in Section 7. We also eliminate the Regge subtraction from the superconvergence type sum rules. Section 8 deals with the numerical evaluation of these sum rules. In Section 9 we review the assumptions made in the derivation and evaluation of the sum rules. We then briefly compare the present sum rules with other works. Finally an appendix contains sum rules satisfied by the scaling functions and their rough estimates based on these.

2. Kinematics

The present work studies the matrix element[2])

$$e_\mu T^\mu = i\int dx\, e^{iQx}\langle p_2 \mid \theta(x^0)[\partial_\nu A_c^\nu(x/2),\quad V_{em}^\mu(-x/2)] \mid p_1\rangle e_\mu, \tag{2.1}$$

representing the 'process' depicted in Fig. 1. $\partial_\nu A_c^\nu(x)$ and $V_{em}^\mu(x)$ denote the divergence of the axial vector current of isospin index c and the electromagnetic current carrying

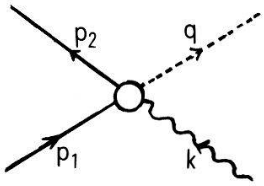

Figure 1
The 'process' $N(p_1) + \gamma(k) \to N(p_2) +$ '$\partial_\nu A_c^\nu$'(q).

momenta q and k respectively. p_1 and p_2 are the momenta of the initial and final nucleon respectively. As a maximal set of four-vectors we choose

$$P = \tfrac{1}{2}(p_1 + p_2),\quad Q = \tfrac{1}{2}(k + q),\quad \Delta = p_2 - p_1 = k - q.$$

It will be convenient for us to work with the following set of independent scalar variables,

$$\nu = P\cdot Q,\quad t = \Delta^2,\quad k^2,\quad q^2.$$

Of course, for the physical pion and photon, $q^2 = m_\pi^2$, $k^2 = 0$.

The most general form of the above amplitude may be written as

$$e_\mu T^\mu = \sum_i U_i \bar{u}(p_2) N_i u(p_1)$$

where

$$\begin{aligned} N_1 &= i\gamma_5 \not{e}\not{k} & N_5 &= -i\gamma_5 \not{e} \\ N_2 &= 2i\gamma_5 P\cdot e & N_6 &= i\gamma_5 \not{k} P\cdot e \\ N_3 &= 2i\gamma_5 q\cdot e & N_7 &= i\gamma_5 \not{k} q\cdot e \\ N_4 &= 2i\gamma_5 k\cdot e & N_8 &= i\gamma_5 \not{k}\cdot k\cdot e. \end{aligned} \tag{2.2}$$

The amplitudes U_i have been shown by Ball [11] to be free of kinematic singularities.

---

[2]) Our convention for metric and γ matrices is that of J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill Book Company, New York, 1965). For other definitions and conventions, we follow Reference [13].

The isospin structure of the amplitudes U_i is

$$U_i = a^{(+)}U_i^{(+)} + a^{(-)}U_i^{(-)} + a^{(0)}U_i^{(0)},$$

where

$$a^{(\pm)} = \tfrac{1}{4}[\tau^c, \tau^3]_\pm, \quad a^{(0)} = \tfrac{1}{2}\tau^c.$$

We shall also use the abbreviated notation

$$\mathcal{N}_i^{(\alpha)} = \bar{u}(p_2)a^{(\alpha)}N_i u(p_1), \quad \alpha = +, -, 0.$$

We shall omit isospin index in an equation when it is valid for any of its values.

Current conservation

$$\partial_\mu V^\mu_{em}(x) = 0,$$

results in the Ward identity

$$\begin{aligned} k_\mu T^\mu &= -\int dx\, e^{ikx}\,\delta(x^0)\langle p_2 \,|\, [\partial_\nu A^\nu_c(0), \quad V^\mu_{em}(-x)] \,|\, p_1\rangle \\ &= -\,\beta H(t) i\bar{u}(p_2)\gamma_5 u(p_1), \end{aligned} \tag{2.3}$$

where, and in the following, the factor β enables us to combine results for isospin $(+)$, $(-)$ and (0) amplitudes in one expression; $\beta^{(+,0)} = 0$, $\beta^{(-)} = 1$, and

$$H(t) = 2mg_A(t) + th_A(t). \tag{2.4}$$

Contributions from Schwinger terms which may be present in the above commutator are assumed to cancel with contact (seagull) contributions which are implied to be present in $e_\mu T^\mu$ to make the retarded amplitude covariant. $g_A(t)$ and $h_A(t)$ are the usual axial vector form factors

$$\langle p_2 \,|\, A^c_\mu(0) \,|\, p_1\rangle = \bar{u}(p_2)\frac{\tau^c}{2}[\gamma_\mu g_A(t) + (p_2 - p_1)_\mu h_A(t)]\gamma_5 u(p_1) \tag{2.5}$$

Equation (2.3) now yields the following constraint among the amplitudes U_i,

$$\begin{aligned} &(U_1 k^2 + U_2 2P\cdot k + U_3 2q\cdot k + U_4 2k^2) i\bar{u}(p_2)\gamma_5 u(p_1) \\ &\qquad + (-\,U_5 + U_6 P\cdot k + U_7 q\cdot k + U_8 k^2) i\bar{u}(p_2)\gamma_5 \not{k} u(p_1) \\ &= -\,\beta H(t) i\bar{u}(p_2)\gamma_5 u(p_1). \end{aligned} \tag{2.6}$$

For non-zero k_μ and q_μ the two kinematic covariants in (2.6) are independent and the above equation splits into two separate constraints,

$$(U_1 + 2U_4)k^2 + U_2 2P\cdot k + U_3 2q\cdot k = -\beta H(t), \tag{2.7}$$

$$-U_5 + U_6 P\cdot k + U_7 q\cdot k + U_8 k^2 = 0. \tag{2.8}$$

Amplitudes for physical pion scattering (we denote them by U_i^π) are recovered from U_i by extracting the residue of the pole at $q^2 = m_\pi^2$. Since H(t) is independent of q^2, the equations of constraint for U_i^π are the same as those of U_i with $H(t)$ replaced by zero.

The crossing symmetry relations for the absorptive part of U_i (denoted as $\overline{U}_i$) under $\nu \to -\nu$ with k^2, q^2, t fixed are

$$\overline{U}_i^{(\pm,0)}(\nu, k^2, q^2, t) = (\pm, +)\zeta_i \overline{U}_i(-\nu, k^2, q^2, t), \tag{2.9}$$

where

$$\zeta_i = +1 \text{ for } i = 3, 5, 7, 8 \text{ and } \zeta_i = -1 \text{ for } i = 1, 2, 6.$$

Also

$$(\bar{U}_1 + 2\bar{U}_4)^{(\pm,0)}(\nu, k^2, q^2, t) = (\pm, +)(\bar{U}_1 + 2\bar{U}_4)^{(\pm,0)}. \tag{2.10}$$

Contributions from the nucleon intermediate state to $U_i^{(\pm,0)}$ are shown in Table I, where we have included terms finite at the nucleon pole along with the pole terms.

Table I
Contribution from nucleon intermediate state to the invariant amplitudes. Finite contributions are retained along with pole contributions. We use the abbreviation $P_\pm = (1/(2P+q)\cdot k) \mp (1/(2P-q)\cdot k)$.

amplitude \ isospin	$(+, 0)$	$(-)$
U_1	$g(F_1^{v,s} + 2mF_2^{v,s})P_+ + g_A F_2^{v,s}$	$g(F_1^v + 2mF_2^v)P_-$
U_2	$-gF_1^{v,s}P_+$	$-gF_1^vP_-$
U_3	$-\frac{1}{2}gF_1^{v,s}P_-$	$-\frac{1}{2}gF_1^vP_+$
U_4	$-\frac{1}{2}gF_1^{v,s} + 2mF_2^{v,s}) - \frac{1}{2}g_AF_2^{v,s}$	$-\frac{1}{2}g(F_1^v + 2mF_2^v)P_-$
U_5	0	$g_AF_1^v + 2gF_2^v$
U_6	$2gF_2^{v,s}P_+$	$2gF_2^vP_-$
U_7	$gF_2^{v,s}P_-$	$gF_2^vP_+$
U_8	0	0

The finite parts are needed for deriving low energy theorems in section 4. The corresponding pole contributions to U_i^π are obtained by replacing $g(q^2) \equiv \frac{1}{2}[2mg_A(q^2) + q^2h_A(q^2)]$ by g_r, the renormalized pion-nucleon coupling constant. $H(t)$ and $g(t)$ are essentially the same function, $H(t) = 2g(t)$. We also note for later use that $H(t)$ satisfies the well-known unsubtracted dispersion relation

$$H(t) = \frac{2f_\pi g_r}{m_\pi^2 - t} + I(t), \qquad I(t) = \frac{1}{\pi}\int_{9m_\pi^2}^{\infty} \frac{ImH(t')}{t' - t}\,dt', \tag{2.11}$$

which is basic to the derivation of Goldberger–Treiman relation, $f_\pi g_r \cong mg_A(0)$, f_π bring the pion decay constant. $F_{12}^{v,s} \equiv F_{1,2}^{v,s}(k^2)$ appearing in Table I are the electromagnetic form factors of the nucleon,

$$\langle p_2 | V_{em}^\mu(0) | p_1\rangle = \bar{u}(p_2)[\gamma^\mu(\tau_3 F_1^v + F_1^s) + i\sigma^{\mu\nu}(p_2 - p_1)_\nu(\tau_3F_2^v + F_2^s)]u(p_1), \tag{2.12}$$

the form factors being so normalized that $F_1^{v,s}(0) = 1$, $F_2^{v,s}(0) = \kappa^{v,s}/2m$, $\kappa^v = 3.70$, $\kappa^s = -0.12$.

Because of relations (2.7) and (2.8), it is possible to eliminate two of the U_i's. The conventional 'gauge invariant' amplitudes result from elimination of U_3 and U_5,

$$e_\mu T^\mu = \sum_i V_i \bar{u}(p_2) O_i u(p_1) - \frac{\beta H(t)}{q\cdot k} i\bar{u}(p_2)\gamma_5 q\cdot e u(p_1).$$

where

$$\begin{aligned} O_1 &= \tfrac{1}{2} i\gamma_5\{\gamma, \gamma\} & O_4 &= 2i\gamma_5(\{\gamma, P\} - m/2\{\gamma, \gamma\}) \\ O_2 &= 2i\gamma_5\{P, q\} & O_5 &= -i\gamma_5\{k, q\} \\ O_3 &= i\gamma_5\{\gamma, q\} & O_6 &= i\gamma_5\{k, \gamma\} \end{aligned}$$

with $\{a, b\} = a\cdot e\, b\cdot k - a\cdot k\, b\cdot e$ and

$$\begin{aligned} V_1 &= U_1 - mU_6 & V_4 &= -\tfrac{1}{2}U_6 \\ V_2 &= U_2/q\cdot k & V_5 &= -(U_1 + 2U_4)/q\cdot k \\ V_3 &= -U_7 & V_6 &= U_8. \end{aligned} \tag{2.13}$$

Thus elimination of U_3 is achieved at the cost of introducing kinematic singularity in V_2 and V_5 at $q\cdot k = 0$. Because $e_\mu T^\mu$ involves $\partial_\mu A_c^\mu$ and not the operator for the physical pion, it cannot be expressed entirely in terms of the 'gauge invariant' amplitudes V_i [12].

Let $\tilde{U}_i(\tilde{V}_i)$ denote amplitudes from which the nucleon pole contribution is subtracted out, i.e. $U_i = U_i^P + \tilde{U}_i$. If κ_2 and κ_5 denote the residues of $\tilde{V}_2$ and $\tilde{V}_5$ at $q\cdot k = 0$, then

$$\kappa_2 = -\tilde{U}_2|_{q\cdot k=0}, \qquad \kappa_5 = -(\tilde{U}_1 + 2\tilde{U}_4)|_{q\cdot k=0}. \tag{2.14}$$

Adler has shown [13] that the requirement, that the kinematic singularity at $q\cdot k = 0$ should not be present in the matrix element in the physical region, yields,

$$\kappa_2 = 0. \tag{2.15}$$

The constraint equation (2.7) at $q\cdot k = 0$ then reads

$$\kappa_5 k^2 + 2g(q^2)F_1^v \beta = \beta H(t). \tag{2.16}$$

This relation will be required in section 4 to convert low energy theorems for U_i to those for V_i.

3. Generalized Bjorken limit

We now determine the behaviour of the invariant amplitudes in the generalized Bjorken limit, in which all the three variables ν, k^2, q^2 tend to infinity maintaining fixed finite ratios amongst themselves. This is achieved by setting [1]

$$Q = Q' + En, \qquad E \to \pm\infty$$

where n_μ is a lightlike vector, $n^2 = 0$ and Q', P, Δ are kept fixed. In terms of the scalar variables the parametrization translates into

$$k^2 = 2\xi_1\nu - \eta_1, \quad q^2 = 2\xi_2\nu - \eta_2,$$

where

$$\xi_1 = (Q + \Delta/2)\cdot n/P\cdot n, \quad \xi_2 = (Q - \Delta/2)\cdot n/P\cdot n,$$

with $\nu \to \infty$ and $t, \xi_1, \xi_2, \eta_1, \eta_2$ fixed. We also define for later use

$$\xi = Q\cdot n/P\cdot n = (\xi_1 + \xi_2)/2, \quad \xi_- = \Delta\cdot n/P\cdot n = \xi_1 - \xi_2, \quad \xi_0 = Q\cdot n/\Delta\cdot n = \xi/\xi_-,$$
$$q\cdot k = 2\xi\nu - \eta', \quad \eta' = (\eta_1 + \eta_2 + t)/2.$$

Under the assumption that the leading light cone behaviour of currents is canonical, i.e. is the same as that for free quark fields of spin 1/2, it is straightforward to determine the absorptive part of the amplitude (2.1) in the generalised Bjorken limit [1, 4, 14],

$$e_\mu T^\mu \xrightarrow[Bj]{} \frac{2im_q}{4\pi}\int_{-\infty}^{+\infty} d\lambda\, e^{i\lambda\xi P\cdot n} C^{\mu\nu}_{c,em}(\lambda n) n_\mu\, e_\nu \equiv e_\mu \overline{T}^\mu_{Bj}, \tag{3.1}$$

where

$$C^{c,em}_{\mu\nu}(x) = C^{(+)}_{\mu\nu}(x) + C^{(-)}_{\mu\nu}(x) + C^{(0)}_{\mu\nu}(x),$$

with

$$C^{(\pm,0)}_{\mu\nu}(x) = \langle p_2 \,|\, \bar{q}(x/2)\gamma_5\gamma_\mu\gamma_\nu\, \Lambda^{(\pm,0)} q(-x/2)(\mp, -)\bar{q}(-x/2)\gamma_\nu\gamma_\mu\gamma_5$$
$$\Lambda^{(\pm,0)} q(x/2) \,|\, p_1\rangle, \tag{3.2}$$

and

$$\Lambda^{(+)} = \frac{1}{8}[\lambda_c, \lambda_3]_+ = \frac{1}{\sqrt{3}}\frac{1}{4}(\lambda_8 + \sqrt{2}\,\lambda_0)\delta_{c3}$$

$$\Lambda^{(-)} = \frac{1}{8}[\lambda_c, \lambda_3]_- = \frac{i}{4}\,\epsilon_{c3k}\lambda_k$$

$$\Lambda^{(0)} = \frac{1}{8}\left[\lambda_c, \frac{1}{\sqrt{3}}\lambda_8\right]_+ = \frac{1}{3}\frac{\lambda_c}{4}.$$

Here $q(x)$ denotes a triplet of quark fields (up, down and strange) and m_q is the average mass of up and down quarks. λ_i are the SU(3) Gell–Mann matrices.

We have now to expand $C^{(\pm,0)}_{\mu\nu}$ in terms of available second rank tensors. Clearly, its symmetric part can be expanded in terms of two tensors, viz,

$$\{\gamma_5 g_{\mu\nu}, A_1\}, \quad \{\gamma_5 g_{\mu\nu} \not{x}, A_2\}, \qquad (\not{x} = x_\mu\gamma^\mu)$$

where and in the following, the second symbol within each bracket denotes the corresponding scalar form factor to appear in the expansion of $C^{(\pm,0)}_{\mu\nu}$. We expand the antisymmetric part in terms of 14 tensors[3]) which may be grouped into 4 classes according to their x-dependence (we omit γ_5 on the left, for short),

---

[3]) The number of linearly independent antisymmetric covariants required to express $C_{\mu\nu}$ is 12 and our choice has 2 in excess. It is not difficult to find 2 relations among them, e.g. one can express the 4-vector $N_\mu = \varepsilon_{\mu\nu\lambda\sigma} p^\nu \Delta^\lambda x^\sigma$ in terms of the set of linearly independent 4-vectors, p_μ, Δ_μ, x_μ and γ_μ. It is then straightforward to convert the identities $N_\mu N_\nu - N_\nu N_\mu = 0$, $\not{x}(N_\mu N_\nu - N_\nu N_\mu) = 0$ into relations among our set of covariants. But we have not succeeded in using such relations to eliminate 2 covariants without introducing kinematic singularities at $x = 0$ or $\Delta = 0$. This situation will, however, not cause any difficulty, as we shall at no stage require the linear independence of our set for $x \neq 0$.

$$\{a_{\mu\nu}, A_{(3)}\}: \quad \{\bar{\sigma}_{\mu\nu}, A_3\}, \quad \{P_\mu \Delta_\nu - P_\nu \Delta_\mu, A_4\}, \quad \{\Delta_\mu \gamma_\nu - \Delta_\nu \gamma_\mu, A_5\}, \quad \{P_\mu \gamma_\nu - P_\nu \gamma_\mu, A_6\};$$

$$\{b_\mu x_\nu - b_\nu x_\mu, A_{(4)}\}: \{P_\mu x_\nu - P_\nu x_\mu, A_7\}, \quad \{\Delta_\mu x_\nu - \Delta_\nu x_\mu, A_8\}, \quad \{\gamma_\mu x_\nu - \gamma_\nu x_\mu, A_9\};$$

$$\{a_{\mu\nu} \not{x}, A_{(5)}\}: \quad \{\bar{\sigma}_{\mu\nu} \not{x}, A_{10}\}, \quad \{(P_\mu \Delta_\nu - P_\nu \Delta_\mu) \not{x}, A_{11}\}, \quad \{(\Delta_\mu \gamma_\nu - \Delta_\nu \gamma_\mu) \not{x}, A_{12}\}, \quad \{(P_\mu \gamma_\nu - P_\nu \gamma_\mu) \not{x}, A_{13}\};$$

$$\{(b_\mu x_\nu - b_\nu x_\mu) \not{x}, A_{(6)}\}: \quad \{(P_\mu x_\nu - P_\nu x_\mu) \not{x}, A_{14}\}, \quad \{(\Delta_\mu x_\nu - \Delta_\nu x_\mu) \not{x}, A_{15}\}, \quad \{(\gamma_\mu x_\nu - \gamma_\nu x_\mu) \not{x}, A_{16}\}.$$

Here $\bar{\sigma}_{\mu\nu} = \gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu$. The above set includes all possible kinematic co-variants leaving aside those which are expressible in terms of this set without introducing kinematic singularity at $x = 0$ or $\Delta = 0$. Now since the local limit ($x = 0$) as well as the forward limit ($\Delta = 0$) of the matrix element $C_{\mu\nu}$ are well defined, it is possible to choose the scalar form factors A_i's which are free of kinematic singularities at $x = 0$ or $\Delta = 0$. Our desired expansion may now be written as

$$C_{\mu\nu}^{(\pm,0)}(x) = \bar{u}(p_2)\gamma_5 \frac{a^{(\pm,0)}}{2} [g_{\mu\nu} A_1^{(\pm,0)} + g_{\mu\nu} \not{x} A_2^{(\pm,0)} + a_{\mu\nu} A_3^{(\pm,0)} + (b_\mu x_\nu - b_\nu x_\mu) A_{(4)}^{(\pm,0)} + a_{\mu\nu} \not{x} A_5^{(\pm,0)} + (b_\mu x_\nu - b_\nu x_\mu) \not{x} A_6^{(\pm,0)}] u(p_1), \tag{3.3}$$

where $A_i^{(\pm,0)} \equiv A_i^{(\pm,0)}(Px, \Delta x/Px, t)$. By this choice of arguments of A_i only $P \cdot x$ depends on x on the light cone. Inserting (3.3) into (3.1) we get

$$e_\mu \bar{T}_{Bj}^\mu(\xi, \xi_-, t) = \frac{im_q}{P \cdot n} \sum_{\alpha = +,-,0} \bar{u}(p_2)\gamma_5 a^{(\alpha)} [g^{\mu\nu} \tilde{A}_1^{(\alpha)} + g^{\mu\nu} \frac{\not{n}}{P \cdot n} \tilde{A}_2^{(\alpha)} + a^{\mu\nu} A_{(3)}^{(\alpha)} + \frac{(b^\mu n^\nu - b^\nu n^\mu)}{P \cdot n} \tilde{A}_{(4)}^{(\alpha)} + a^{\mu\nu} \frac{\not{n}}{P \cdot n} \tilde{A}_{(5)}^{(\alpha)} + \frac{(b^\mu n^\nu - b^\nu n^\mu)}{(P \cdot n)^2} \not{n} \tilde{A}_{(6)}^{(\alpha)}] u(p_1) n_\mu e_\nu \tag{3.4}$$

where

$$\tilde{A}_i \equiv \tilde{A}_i(\xi, \xi_-, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d(P \cdot x)\, e^{i\xi P \cdot x} K(Px) A_i(Px, \Delta x/Px, t) \tag{3.5}$$

with $K(Px)$ equal to 1, Px, $(P \cdot x)^2$ for $i = 1, (3)$; $i = 2, (4), (5)$; $i = (6)$ respectively.

In the generalized Bjorken limit, the invariant decomposition of the absorptive part of the amplitude (2.2) may be written as

$$e_\mu \bar{T}^\mu = \sum_{\alpha = +,-,0} i\bar{u}(p_2)\gamma_5 a^{(\alpha)} [\bar{U}_1^{(\alpha)} E \not{n} \not{e} + 2\bar{U}_2^{(\alpha)} P \cdot e + 2(\bar{U}_3 + \bar{U}_4)^{(\alpha)} E n \cdot e - \bar{U}_5^{(\alpha)} \not{e} + \bar{U}_6^{(\alpha)} \not{n} P \cdot e + (\bar{U}_7 + \bar{U}_8)^{(\alpha)} E^2 \not{n} \not{e} + \left(\frac{\bar{U}_1}{2} - \bar{U}_3 + \bar{U}_6 + \frac{m}{2}(\bar{U}_7 - \bar{U}_8)\right)^{(\alpha)} \Delta \cdot e - \tfrac{1}{2}(\bar{U}_7 - \bar{U}_8)^{(\alpha)} E \Delta \cdot e] u(p_1). \tag{3.6}$$

The scaling limit of the invariant amplitudes are now obtained by equating (3.6) and (3.4) (omitting isospin index for short).

$$
\begin{aligned}
&\bar{U}_1 \to \bar{S}_1/\nu, && \bar{S}_1 = \tfrac{1}{2}m_q(-2\tilde{A}_3 + \xi_-\tilde{A}_{12} + \tilde{A}_{13})\\
&\bar{U}_2 \to \bar{S}_2, && \bar{S}_2 = -\tfrac{1}{4}m_q\xi_-\tilde{A}_4\\
&\bar{U}_3 \to \bar{S}_3, && \bar{S}_3 = -\tfrac{1}{4}m_q\tilde{A}_4\\
&\bar{U}_4 \to \bar{S}_4, && \bar{S}_4 = \tfrac{1}{4}m_q\tilde{A}_4\\
&\bar{U}_5 \to \bar{S}_5, && \bar{S}_5 = \tfrac{1}{2}m_q(\xi_-\tilde{A}_5 + \tilde{A}_6)\\
&\bar{U}_6 \to \bar{S}_6/\nu, && \bar{S}_6 = -\tfrac{1}{2}m_q(\tilde{A}_6 + \xi_-\tilde{A}_{11})\\
&\bar{U}_7 \to \bar{S}_7/\nu, && \bar{S}_7 = \tfrac{1}{2}m_q(\tilde{A}_5 - \tilde{A}_{11})\\
&\bar{U}_8 \to \bar{S}_8/\nu, && \bar{S}_8 = -\tfrac{1}{2}m_q(\tilde{A}_5 - \tilde{A}_{11})\\
&\bar{U}_3 + \bar{U}_4 \to \bar{S}_{34}/\nu, && \bar{S}_{34} = \tfrac{1}{4}m_q(\tilde{A}_1 + 2\tilde{A}_3 + \tilde{A}_7 + \xi_-\tilde{A}_8)\\
&\bar{U}_7 + \bar{U}_8 \to \bar{S}_{78}/\nu^2, && \bar{S}_{78} = \tfrac{1}{2}m_q(\tilde{A}_2 + \tilde{A}_9 + 2\tilde{A}_{10} + \tilde{A}_{14} + \xi_-\tilde{A}_{15}).
\end{aligned}
\tag{3.7}
$$

It is easy to verify that the relations among scaling functions (both absorptive and retarded) given by current conservation constraints (2.7) and (2.8) are satisfied identically.

4. Low energy theorems

Since we intend to make use of low energy theorems [5, 6] to write down light cone sum rules, it is convenient here to rederive them for the amplitudes U_i. The use of U_i rather than V_i will allow us to obtain two new sum rules in the next section.

We evaluate the amplitude $e_\mu T^\mu$ as $q_\mu \to 0$ through the Ward identity

$$
\begin{aligned}
e_\mu T^\mu = & -i\int dx\, e^{iqx}\delta(x^0)\langle p_2 \mid [A_c^0(x), V_{em}^\mu(0)] \mid p_1\rangle e_\mu \\
& + q_\nu \int dx\, e^{iqx}\theta(x^0)\langle p_2 \mid [A_c^\nu(x), V_{em}^\mu(0)] \mid p_1\rangle e_\mu
\end{aligned}
\tag{4.1}
$$

The term with the equal time commutator, in the limit $q_\mu \to 0$, is

$$
\varepsilon_{c3\alpha}\langle p_2 \mid A_\alpha^\mu \mid p_1\rangle = i\bar{u}(p_2)\gamma_5 a^{(-)}(\not{e}g_A(t) - k\cdot e h_A(t))u(p_1). \tag{4.2}
$$

The term proportional to q_μ receives contribution from only the nucleon intermediate state in the limit $q_\mu \to 0$. These (both pole and finite) contributions may be obtained from Table 1. Collecting all the results, (4.1) becomes

$$
\begin{aligned}
e_\mu T^\mu\big|_{q_\mu\to 0} = & \text{ pole terms} + i\bar{u}(p_2)\gamma_5 a^{(-)}(\not{e}g_A(k^2) - k\cdot e h_A(k^2))u(p_1)\\
& + g_A(0)i\bar{u}(p_2)\gamma_5[\{F_2^s(k^2)a^{(0)} + F_2^v(k^2)a^{(+)}\}(\not{e}\not{k} - k\cdot e)\\
& - \{F_1^v(k^2) + 2mF_2^v(k^2)a^{(-)}\not{e}]u(p_1)\\
= & \text{ pole terms} + g_A(0)(F_2^s(k^2)\mathcal{N}_2^{(0)} + F_2^v(k^2)\mathcal{N}_2^{(+)}) - \tfrac{1}{2}h_A(k^2)\mathcal{N}_4^{(-)}\\
& + [g_A(0)\{F_1^v(k^2) + 2mF_2^v(k^2)\} - g_A(k^2)]\mathcal{N}_5^{(-)}
\end{aligned}
\tag{4.3}
$$

Let $\tilde{U}_i$ denote the non-pole part of U_i in the limit $q_\mu \to 0$: $U_i = U_i^P + \tilde{U}_i$. Being free of kinematic singularity at $q \cdot k = 0$, $\tilde{U}_i$ are finite at $q_\mu = 0$. From (2.2) we get

$$e_\mu T^\mu \big|_{q_\mu \to 0} = \sum_{i,a} U_i^{P,(a)} \mathcal{N}_i^{(a)} + \sum_{i,a} \tilde{U}_i^{(a)} \mathcal{N}_i^{(a)}$$

$$= \text{pole terms} + \sum_a \Bigg[(\tilde{U}_1 - m\tilde{U}_6 + \tilde{U}_2)^{(a)} \mathcal{N}_2^{(a)}$$

$$+ \left(\frac{\tilde{U}_1}{2} + \tilde{U}_4 - m\tilde{U}_8 \right) \mathcal{N}_4^{(a)} + \tilde{U}_5^{(a)} \mathcal{N}_5^{(a)} \Bigg] \tag{4.4}$$

The desired low energy theorems now follow from equating (4.3) and (4.4):

$$(\tilde{U}_1 - m\tilde{U}_6 + \tilde{U}_2)^{(+,0)} = g_A(0) F_2^{v,s}(k^2) \tag{4.5}$$

$$\left(\frac{\tilde{U}^1}{2} + \tilde{U}_4 - m\tilde{U}_8 \right)^{(-)} = -\tfrac{1}{2} h_A(k^2) \tag{4.6}$$

$$\tilde{U}_5^{(-)} = g_A(0)\{F_1^v(k^2) + 2mF_2^v(k^2)\} - g_A(k^2) \tag{4.7}$$

at $q_\mu = 0$. As a check, we may verify that these expressions satisfy the constraint equation (2.6) among U_i's in the limit $q_\mu \to 0$. The combinations of U_i's entering the low energy theorems do not actually possess any nucleon pole contribution. Hence the tilde ($\sim$) on them may be dropped.

Recalling (2.13), (2.14) and (2.15), the low energy theorems may more conventionally be expressed in terms of V_i:

$$\tilde{V}_1^{(+,0)} = (\tilde{U}_1 - m\tilde{U}_6)^{(+,0)} = g_A(0) F_2^{v,s}(k^2) \tag{4.8}$$

$$-2m\tilde{V}_6^{(-)} = -2m\tilde{U}_8^{(-)} = -h_A(k^2) + \kappa_5^{(-)} \tag{4.9}$$

$$k^2 \tilde{V}_6^{(-)} = U_5^{(-)} - 2gF_2^v(k^2) = g_A(0) F_1^v(k^2) - g_A(k^2) \tag{4.10}$$

With the help of (2.8) and (2.16) it is easy to recognize the identity of (4.9) and (4.10). We finally get the set of low energy theorems by collecting all evaluations of $\tilde{U}_i$ as $q_\mu \to 0$:

$$(\tilde{U}_1 - m\tilde{U}_6)^{(+,0)} = g_A(0) F_2^{v,s}(k^2) \tag{4.11}$$

$$\tilde{U}_8^{(-)} = -\frac{1}{k^2}[g_A(k^2) - g_A(0) F_1^v(k^2)] \tag{4.12}$$

$$\tilde{U}_2^{(+,0)} = -\kappa_2^{(+,0)} = 0 \tag{4.13}$$

$$(\tilde{U}_1 + 2\tilde{U}_4)^{(-)} = -\kappa_5^{(-)} = -h_A(k^2) - \frac{2m}{k^2}[g_A(k^2) - g_A(0) F_1^v(k^2)] \tag{4.14}$$

5. Light cone sum rules

We now write down the light cone dispersion relations (LCDR) and light cone sum rules (LCSR) [1, 2] which are satisfied by the invariant amplitudes. These embody the statement of causality of the commutator in (2.1) and may be looked upon as generalization of fixed mass dispersion relations and sum rules. However, contrary to the fixed mass case where the asymptotics is determined by the leading

Regge singularities in the crossed channel, the asymptotic behaviour relevant for writing LCDR and LCSR is that in the generalized Bjorken limit, which may be thought of as originating from fixed poles in the complex angular momentum plane of the crossed channel partial waves of the current amplitude.

Suppose an amplitude $A(\nu, k^2 = 2\xi_1\nu - \eta_1, q^2 = 2\xi_2\nu - \eta_2, t)$ which is free of kinematic singularities in ν, behaves in the generalized Bjorken limit as

$$A(\nu, k^2, q^2, t) \xrightarrow[Bj]{} S_1(\xi, \xi_-, t) \tag{5.1}$$

where S_1 is the retarded scaling function

$$S_1(\xi, \xi_-, t) = P\int_{-1}^{+1} \frac{\bar{S}_1(\xi', \xi_-, t)}{\xi' - \xi}\, d\xi' \tag{5.2}$$

This behaviour then allows us to write a LCDR for A,

$$A(\nu, q^2, k^2, t) = \int_{-\infty}^{+\infty} \frac{d\nu'}{\nu' - \nu}\, \overline{A}(\nu', q'^2, k'^2, t) + S_1(\xi, \xi_-, t) \tag{5.3}$$

where a symmetrical integration over ν' is implied. If, however, an amplitude $B(\nu, k^2, q^2, t)$ behaves as

$$B(\nu, k^2, q^2, t) \xrightarrow[Bj]{} S_2(\xi, \xi_-, t)/\nu, \tag{5.4}$$

then we not only have a LCDR,

$$B(\nu, k^2, q^2, t) = \int_{-\infty}^{+\infty} \frac{d\nu'}{\nu' - \nu}\, \overline{B}(\nu', q'^2, k'^2, t) \tag{5.5}$$

but also a LCSR,

$$\int_{-\infty}^{+\infty} d\nu B(\nu, q^2, k^2, t) = -\, S_2(\xi, \xi_-, t) \tag{5.6}$$

a symmetrical integration being again implied in (5.6).

The path of integration in the domain of the variables (ν, q^2, k^2) appearing in the above integrals is specified by the parameters $\xi_1, \xi_2, \eta_1, \eta_2$. Since the low energy theorems predict the amplitudes only at $q_\mu = 0$, i.e. $\nu = q^2 = q\cdot k = 0$, we must choose the path of integration such that it passes through this point in order to make use of them. This is achieved by choosing $\eta_2 = 0$, $\eta_1 = -t$, i.e. with

$$k^2 = 2\xi_1\nu + t, \quad q^2 = 2\xi_2\nu, \quad q\cdot k = 2\xi\nu.$$

It is then easy to incorporate the low energy theorems in the LCDRs to obtain the following low energy sum rules:

$$\int_{-\infty}^{+\infty} \frac{d\nu}{\nu}\, (\overline{U}_1 - m\overline{U}_6)^{(+,0)}(\nu, q^2, k^2, t) = g_A(0) F_2^{(v,s)}(t) \tag{5.7}$$

$$\int_{-\infty}^{+\infty} \frac{d\nu}{\nu}\, \overline{U}_8^{(-)}(\nu, q^2, k^2, t) = -\frac{1}{t}[g_A(t) - g_A(0)F_1^v(t)] \tag{5.8}$$

$$\int_{-\infty}^{+\infty} \frac{d\nu}{\nu}\, \overline{U}_2^{(+,0)}(\nu, q^2, k^2, t) = -\, S_2^{(+,0)}(\xi, \xi_-, t) \tag{5.9}$$

$$\int_{-\infty}^{+\infty} \frac{dv}{v} (\bar{U}_1 + 2\bar{U}_4)^{(-)}(v, q^2, k^2, t) = -\frac{1}{t} [2mg_A(t) + th_A(t)$$

$$- 2mg_A(0)F_1^v(t)] - 2S_4(\xi, \xi_-, t) \qquad (5.10)$$

The presence of retarded scaling functions in (5.9) and (5.10) is inferred from (3.7). Note that the dispersion integrals in these sum rules do not include any nucleon pole term.

Equations (3.7) also show that the amplitudes satisfying zeroth moment LCSRs with scaling functions determined by the leading light cone singularity are U_1, $U_3 + U_4$, U_6, U_7 and U_8. We thus have the sum rules

$$\int_{-\infty}^{+\infty} dv\bar{U}_i^{(\pm,0)}(v, k^2, q^2, t) = g(q^2)R_i^{(\pm,0)}(k^2) - S_i^{(\pm,0)}(\xi, \xi_-, t),$$

$$i = 1, (3+4)^{4)}, 6, 7, 8 \qquad (5.11)$$

The first term on the right is the residue of the nucleon pole (R_i involves only the nucleon electromagnetic form factors and may be read off from Table I). It is also possible to write the first moment LCSR for the combination $(U_7 + U_8)^{(\pm,0)}$ with known scaling function. But we do not consider it in the present work as it is likely to be sensitive to errors in the evaluation.

6. Elimination of pion contribution

A remarkable distinction shows up between the light cone (variable mass) dispersion relations (and sum rules) and the fixed mass ones in the way they treat the t channel singularities. The (fixed t) fixed mass dispersion integrals do not get any contribution directly from the t channel singularities, but shows up their presence by the divergence of the integral as the singularity in t is approached; the integral thus requires analytic continuation to cross the singularity. On the other hand, the (fixed t) light cone dispersion integrals get contribution from t channel singularities for all value of t through the mass dependence of form factors and do not require analytic continuation to cross such a singularity.

The pion pole in t channel is unique in its proximity to the physical region, its largeness of contribution and its being the only surviving contribution in t channel in the chiral limit. Also its contribution to the amplitude and the scaling function is essentially known [4, 15]. This motivates us to eliminate this singularity from our light cone sum rules and view the resulting ones as constraint among other unknown or less known singularities. This task we wish to carry out below. The pion pole elimination also serves as a partial check on the correctness of our sum rules. Moreover, it will carry out in part the Regge subtraction to be performed in the next section.

We first identify the π pole contribution to the absorptive part of the full amplitude (Fig. 2). Denoting pion contribution by the subscript π, we have

$$e_\mu \bar{T}_\pi^\mu = \frac{ig_r\bar{u}(p_2)\gamma_5\tau^\alpha u(p_1)}{m_\pi^2 - t} \overline{W}^{c,\alpha} \qquad (6.1)$$

4) $U_{3+4} = U_3 + U_4$.

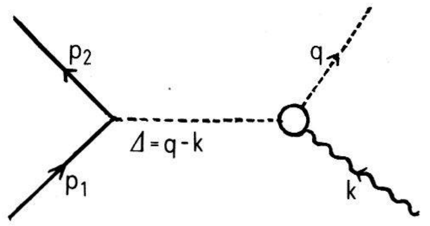

Figure 2
The pion pole in t channel.

where

$$\overline{W}^{c,\alpha} = \frac{1}{2\pi}\int dx\, e^{iQx}\langle 0 \,|\, [\partial^{\nu} A_{\nu}^{c}(x/2), V^{\mu}(-x/2)] \,|\, \pi^{\alpha}, \Delta = q - k\rangle e_{\mu}$$

$$= (q\cdot e\overline{W}_3 + k\cdot e\overline{W}_4)\frac{i}{4}\varepsilon^{c3\alpha} \tag{6.2}$$

so that

$$e_{\mu}\overline{T}_{\pi}^{\mu} = \frac{ig_r\bar{u}(p_2)\gamma_5\frac{1}{8}[\tau^c, \tau^3]u(p_1)}{m_{\pi}^2 - t}(q\cdot e\overline{W}_3 + k\cdot e\overline{W}_4) \tag{6.3}$$

Referring to the invariant decomposition of the amplitude (2.2) we get

$$\overline{U}_{3,\pi}^{(-)} = \frac{1}{4}\frac{g_r}{m_{\pi}^2 - t}\overline{W}_3, \quad \overline{U}_{4,\pi}^{(-)} = \frac{1}{4}\frac{g_r}{m_{\pi}^2 - t}\overline{W}_4 \tag{6.4}$$

The properties of the vertex functions W_3 and W_4 are investigated in detail in Reference (15). We briefly describe those which we shall require. In the Bjorken limit (6.2) becomes

$$(\overline{W}_3 + \overline{W}_4)n\cdot eE\frac{i}{4}\varepsilon^{c3\alpha}\xrightarrow[Bj]{}\frac{2im_q}{4\pi}\int_{-\infty}^{+\infty} d\lambda\, e^{i\lambda Q\cdot n}D^{\mu\nu}n_{\mu}n_{\nu} \tag{6.5}$$

where

$$D_{\mu\nu}(x)$$
$$= \langle 0 \,|\, \bar{q}(x/2)\gamma_5\gamma_{\mu}\gamma_{\nu}\Lambda^{(-)}q(-x/2) + \bar{q}(-x/2)\gamma_{\nu}\gamma_{\mu}\gamma_5\Lambda^{(-)}q(x/2) \,|\, \pi^{\alpha}, \Delta = q - k\rangle \tag{6.6}$$

With the decomposition

$$iD_{\mu\nu}(x) = \frac{i}{4}\varepsilon^{c3\alpha}\frac{2f_{\pi}m_{\pi}^2}{m_q}[g_{\mu\nu}d_3(\Delta\cdot x) + (x_{\mu}\Delta_{\nu} - x_{\nu}\Delta_{\mu})\,d_4(\Delta\cdot x)] \tag{6.7}$$

equation (6.5) becomes

$$(\overline{W}_3 + \overline{W}_4)\xrightarrow[Bj]{}\frac{G(\xi_0)}{E\Delta\cdot n} \tag{6.8}$$

where

$$G(\xi_0) = 2f_{\pi}m_{\pi}^2[\tilde{d}_3(\xi_0) - \tilde{d}_4(\xi_0)]. \tag{6.9}$$

Here $\tilde{d}_i(\xi_0)$ are the Fourier transform of $d_i(\Delta\cdot x)$,

$$\tilde{d}_i(\xi_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d(\lambda\Delta\cdot n)\, e^{i\xi_0\Delta\cdot n} d_i(\lambda\Delta\cdot n), \quad i = 3, 4 \tag{6.10}$$

satisfying the sum rules

$$\int_{-1/2}^{+1/2} d\xi_0 \tilde{d}_3(\xi_0) = 1, \qquad \int_{-1/2}^{+1/2} d\xi_0 \tilde{d}_4(\xi_0) = 0$$
$$\int_{-1/2}^{+1/2} d\xi_0(\xi_0 + \tfrac{1}{2})\tilde{d}_3(\xi_0) = \tfrac{1}{2}, \quad \int_{-1/2}^{+1/2} d\xi_0(\xi_0 + \tfrac{1}{2})\tilde{d}_4(\xi_0) = \frac{1}{6}\left(-1 + \frac{m_q^2}{m_\pi^2}\right) \tag{6.11}$$

Current conservation applied to (6.2) gives

$$q\cdot k\overline{W}_3 + k^2\,\overline{W}_4 = 0. \tag{6.12}$$

Defining now the scaling functions for W_i,

$$\overline{W}_i \xrightarrow[Bj]{} \frac{\overline{G}_i(\xi_0)}{E\Delta\cdot n}, \quad i = 3, 4 \tag{6.13}$$

(6.8) and (6.12) yield

$$\overline{G}_3(\xi_0) = (2\xi_0 + 1)\overline{G}(\xi_0)$$
$$\overline{G}_4(\xi_0) = -2\xi_0\overline{G}(\xi_0). \tag{6.14}$$

Clearly W_i satisfy a LCDR and a LCSR. In terms of the variables

$$\nu_1 = \Delta\cdot Q = \xi_-\nu - \zeta,$$
$$k^2 = 2\xi_1\nu - \eta_1 = (2\xi_0 + 1)\nu_1 - \zeta_1$$
$$q^2 = 2\xi_2\nu - \eta_2 = (2\xi_0 - 1)\nu_1 - \zeta_2$$

these are

$$W_i(\nu_1, k^2, q^2) = \int_{-\infty}^{+\infty} \frac{d\nu_1'}{\nu_1' - \nu_1}\, \overline{W}_i(\nu_1', q'^2, k'^2), \quad i = 3, 4 \tag{6.15}$$

$$\int d\nu_1\, \overline{W}(\nu_1, k^2, q^2) = -\int_{-1/2}^{+1/2} d\xi_0'\, \frac{\overline{G}(\xi_0')}{\xi_0' - \xi_0}, \quad i = 3, 4 \tag{6.16}$$

Comparing (3.6) and (6.4) we find that of all the scaling functions only the one for $(U_3 + U_4)^{(-)}$ possess the pion pole, which from (6.8) is

$$S_{34} = \frac{1}{4}\frac{g_r}{m_\pi^2 - t}\, G(\xi_0) = \frac{1}{4}\frac{g_r}{m_\pi^2 - t}\int_{-1/2}^{+1/2} d\xi_0'\, \frac{\overline{G}(\xi_0')}{\xi_0' - \xi_0}. \tag{6.17}$$

We also note the low energy theorem related to pion decay which is obtainable from the Ward identity for the full amplitude $e_\mu W^\mu$. As $q_\mu \to 0$, we have

$$\tfrac{1}{4} W_4(q^2 = 0, q\cdot\Delta = 0) = -f_\pi. \tag{6.18}$$

We are now in a position to identify the π pole contribution in t channel in our sum rules. It is now clear that only the sum rules for $(U_1 + 2U_4)^{(-)}$ and $(U_3 + U_4)^{(-)}$

receive π pole contributions. Consider first the one for $(U_1 + 2U_4)^{(-)}$ given by (5.10). Equation (6.4) shows that π contributes to the left hand side an amount

$$\frac{1}{4}\frac{g_r}{m_\pi^2 - t}\cdot 2\int \frac{dv}{v}\,\overline{W}_4(v', q'^2, k'^2) = \frac{1}{2}\frac{g_r}{m_\pi^2 - t}\,W_4(v = q^2 = 0, k^2 = t) \qquad (6.19)$$

where we use (6.15). Again the π contribution to the right hand side in $h_A(t)$ is

$$-\frac{2g_r f_\pi}{m_\pi^2 - t}$$

which, by the low energy theorem (6.18), is identical to (6.19). Cancellation of π contribution then leads to

$$\int_{-\infty}^{+\infty}\frac{dv}{v}(U_1 + 2U_4')^{(-)}(v, q^2, k^2, t) = \frac{1}{t}\,[2mg_A(0)F_1^v(t) - 2f_\pi g_r - I(t)] - 2S_4 \qquad (6.20)$$

where $I(t)$ is defined in (2.11). The removal of π contribution is denoted by a prime.

Next, the t channel π contribution on the left of the sum rule (5.11) for $(U_3 + U_4)^{(-)}$ is

$$\begin{aligned}\frac{1}{4}\frac{g_r}{m^2 - t}&\int_{-\infty}^{+\infty} dv(\overline{W}_3 + \overline{W}_4)(v, k^2, q^2, t)\\ &= \frac{1}{4}\frac{g_r}{m_\pi^2 - t}\frac{1}{\xi_-}\int_{-\infty}^{+\infty} dv_1(\overline{W}_3 + \overline{W}_4)(v_1, k^2(v_1), q^2(v_1), t)\\ &= -\frac{1}{4}\frac{g_r}{m_\pi^2 - t}\frac{1}{\xi_-}\int_{-1/2}^{+1/2} d\xi_0'\,\frac{\overline{G}(\xi_0')}{\xi_0' - \xi_0}\end{aligned}$$

which is indeed the negative of the pion part of the scaling function for $(U_3 + U_4)^{(-)}$ as may be seen from (6.17). After elimination of π pole contribution the sum rule becomes

$$\int dv(\overline{U}_3' + \overline{U}_4')^{(-)}(v, q^2, k^2, t) = g(q^2)R_{34}^{(-)} - S_{34}'(\xi, \xi_-, t) \qquad (6.21)$$

7. Fixed mass limit

Since the light cone sum rules written above do not allow a ready evaluation of the dispersion integrals occurring in it, we have to cast the dispersion integrals in a form where the masses of both the axial vector divergence and the photon are held fixed, i.e. we have to interchange the limits $\xi_1, \xi_2 \to 0$ with the integral over v [4]. The possibility of this interchange is governed by the singularities in the J-plane of the crossed channel partial waves of the fixed mass amplitude under consideration.

Calling any one of the U_i's or their combinations appearing in the sum rule as U, let

$$\overline{U} \xrightarrow[R]{} \overline{U}_R = \sum_{\bar{\alpha}_j \geqslant \alpha_0} \gamma_j(q^2 = -\eta_2, k^2 = -\eta_1, t)\, v^{\bar{\alpha}_j(t)}$$

be its behaviour in the Regge limit. (Note that $\bar{\alpha}_j(t)$ may differ from the actual Regge

Table II
High energy behaviour of invariant amplitudes from usually accepted Regge poles and fixed poles.

	Regge behaviour			Fixed pole behaviour	
amplitude \ isospin	(0)	(+)	(−)	(0, +)	(−)
U_1	$\nu^{\alpha_\rho - 1}$	$\nu^{\alpha_\omega - 1}$	$\nu^{\alpha_{A_2} - 1}$	ν^{-2}	ν^{-1}
U_2	$\nu^{\alpha_\rho - 1}$, $\nu^{\alpha_B - 1}$	$\nu^{\alpha_\omega - 1}$	$\nu^{\alpha_{A_2} - 1}$, $\nu^{\alpha_\pi - 1}$, $\nu^{\alpha_{A_1} - 2}$	ν^{-2}	ν^{-1}
U_3	ν^{α_ρ}, ν^{α_B}	ν^{α_ω}	$\nu^{\alpha_{A_2}}$, $\nu^{\alpha_{A_1} - 1}$, ν^{α_π}	ν^{-1}	constant
U_4	ν^{α_ρ}, ν^{α_B}	ν^{α_ω}	$\nu^{\alpha_{A_2}}$, $\nu^{\alpha_{A_1} - 1}$, ν^{α_π}	ν^{-1}	constant
U_5	ν^{α_ρ}	ν^{α_ω}	$\nu^{\alpha_{A_2}}$, $\nu^{\alpha_{A_1} - 1}$	ν^{-1}	constant
U_6	$\nu^{\alpha_\rho - 1}$	$\nu^{\alpha_\omega - 1}$	$\nu^{\alpha_{A_2} - 1}$, $\nu^{\alpha_{A_1} - 2}$	ν^{-2}	ν^{-1}
U_7	$\nu^{\alpha_\rho - 2}$	$\nu^{\alpha_\omega - 2}$	$\nu^{\alpha_{A_1} - 1}$, $\nu^{\alpha_{A_2} - 2}$	ν^{-1}	ν^{-2}
U_8	$\nu^{\alpha_\rho - 2}$	$\nu^{\alpha_\omega - 2}$	$\nu^{\alpha_{A_1} - 1}$, $\nu^{\alpha_{A_2} - 2}$	ν^{-1}	ν^{-2}

trajectory function by 1 or 2 units. See Table II.) It includes asymptotic pieces from both Regge poles and cuts (in the latter case γ_j may also have *lnν* dependence) above α_0 (to be specified below). Then the above light cone sum rules rewritten for the amplitude

$$\bar{U}' = \bar{U} - \bar{U}_R,$$

will allow us to pass on to the fixed mass limit. Clearly, for the low energy sum rules (employing dispersion relation) $\alpha_0 = 0$ and for the superconvergence type sum rules, $\alpha_0 = -1$. The general form of the subtracted sum rules in the fixed mass limit are then

$$\int_{-\infty}^{+\infty} \frac{d\nu}{\nu} \bar{U}'(\nu, k^2 = t, q^2 = 0, t) = U(\nu = q \cdot k = 0, k^2 = t, t) + S'(t), \quad (7.1)$$

and

$$\int_{-\infty}^{+\infty} d\nu \bar{U}'(\nu, k^2 = -\eta_1, q^2 = -\eta_2, t) = g(q^2 = -\eta_2) R(k^2 = -\eta_1) - S'(t), \quad (7.2)$$

for the two types respectively. $S'(t)$ is the retarded scaling function for U'.

Table II shows the high energy behaviour [16] of the invariant (fixed mass) amplitudes in terms of the usually accepted Regge pole trajectories. (The table also includes the asymptotic behaviour of amplitudes in terms of a fixed pole which may exist at the highest right signature non-sense J value in the crossed channel partial wave expansion of the amplitude.) It is now clear that the low energy sum rules (5.7) and (5.8) employing LCDR for $(U_1 - mU_6)^{(+,0)}$ and $U_8^{(-)}$ and the superconvergence type sum rules for $U_7^{(+,0)}$ and $U_8^{(+,0)}$ do not require any Regge subtraction for $t \simeq 0$ in the fixed mass limit, while all others do.

We now use the PCAC choice of pion field [17]

$$\partial_\mu A_c^\mu(x) = f_\pi m_\pi^2 \phi_c(x),$$

and the 'exact' Goldberger–Treiman relation

$$f_\pi g_r(0) = m g_A(0),$$

where $g_r(0)$ in the off-shell ($m_\pi^2 = 0$) πN coupling constant. This allows us to convert the amplitude involving $\partial_\mu A^\mu$ into off-shell π-electroproduction amplitude (denoted by superscript π). Equations (7.1) and (7.2) then become

$$2\int_{mm_\pi + m_\pi^2/2}^{\infty} \frac{d\nu}{\nu}\left[\frac{mg_A(0)}{g_r}\frac{g_r}{g_r(0)}\overline{U}^\pi(\nu, q^2 = 0, k^2 = t, t) - \sum_{\bar{\alpha}_j \geq 0} \gamma_j(q^2 = 0, k^2 = t, t)\nu^{\bar{\alpha}_j(t)}\right] = U(\nu = q\cdot k = 0, k^2 = t, t) + S'(t) \quad (7.3)$$

and

$$2\int_{mm_\pi + \frac{1}{4}(2m_\pi^2 + t + \eta_1)}^{\infty} d\nu\left[\frac{mg_A(0)}{g_r}\frac{g_r}{g_r(0)}\overline{U}^\pi(\nu, k^2 = -\eta_1, q^2 = -\eta_2, t) - \sum_{\bar{\alpha}_j \geq -1} \gamma_j(k^2 = -\eta_1, q^2 = 0, t)\nu^{\bar{\alpha}_j(t)}\right]$$
$$= mg_A(0)R(k^2 = -\eta_1) - S'(t) \quad (7.4)$$

We shall employ Adler's model [18] for off-shell continuation to evaluate $[g_r/g_r(0)]\overline{U}^\pi(\nu, q^2 = 0)$ (see Section 8).

The sum rules (7.3) and (7.4), as it stands, are not very useful in practice. As mentioned in the Introduction, the available Regge fits for photo-production amplitudes do not at all extrapolate the low energy region (in the sense of FESR) to the accuracy demanded by the above sum rules. If one insists on using the available Regge parametrizations to evaluate these sum rules, the large discrepancy between the two sides of the corresponding FESR will effectively show up as a huge contribution for the retarded scaling function and our estimate for the latter will turn out to be hopelessly wrong.

To eliminate the Regge piece from the superconvergence type sum rules, we just note that a similar sum rule for on-shell pion scattering amplitude reads

$$2\int_{mm_\pi + \frac{1}{4}(m_\pi^2 + t + \eta_1)}^{\infty} d\nu\left[\overline{U}_\pi(\nu, k^2 = -\eta_1, q^2 = m_\pi^2, t) - \sum_{\bar{\alpha}_j \geq -1} \beta_i(k^2 = -\eta_1, q^2 = m_\pi^2, t)\nu^{\bar{\alpha}_j(t)}\right] = g_r R(k^2 = -\eta_1) \quad (7.5)$$

which may be obtained directly or by equating the residue of the pole at $q^2 = -\eta_2 = m_\pi^2$ on both sides of (7.2). β_i denotes the Regge residues for physical pion scattering. Assuming a dispersion relation for $\gamma_i(q^2)$ in the unsubtracted form (since it is essentially the matrix element of $\partial_\mu A^\mu$) and that the absorptive part is resonance (say A_1)

dominated, we may write

$$\gamma_j(q^2) = \frac{f_\pi m_\pi^2}{m_\pi^2 - q^2}\beta_j(m_\pi^2) + \frac{C_j}{m_{A_1}^2 - q^2} \tag{7.6}$$

(where C_j is a constant), which at $q^2 = 0$ is

$$\gamma_j(q^2 = 0) = f_\pi \beta_j(m_\pi^2)\left(1 + \frac{\delta}{f_\pi m_{A_1}^2}\right). \tag{7.7}$$

Here $\delta(\equiv C_j/\beta_j)$ is assumed to be the same for all Regge trajectories contributing to a given amplitude. We further assume $\delta/f_\pi m_{A_1}^2 \ll 1$, and eliminate the Regge piece between (7.4) and (7.5) to get

$$\begin{aligned} 2\Bigg[f_\pi \int_{mm_\pi + \frac{1}{4}(m_\pi + t + \eta_1)}^{\infty} d\nu \bar{U}_\pi(\nu, k^2 = -\eta_1, q^2 = m_\pi^2, t) \\ - \frac{mg_A(0)}{g_r}\int_{mm_\pi + \frac{1}{4}(2m_\pi^2 + t + \eta_1)}^{\infty} \frac{g_r}{g_r(0)}\bar{U}_\pi(\nu, k^2 = -\eta_1, q^2 = 0, t)\Bigg] \\ = (f_\pi g_r - mg_A(0))R + S(t). \end{aligned} \tag{7.8}$$

It should be noted that the retarded scaling functions in these sum rules, if non-zero, amount to fixed poles in the J-plane of the current amplitude. Our sum rules thus assume the behaviour of the current commutator on the light cone as the only source of fixed pole contribution.

Clearly in the chiral symmetry limit, $m_q \sim m_\pi^2 \to 0$, each term in the sum rule (7.8) individually goes to zero. We note that a similar method of elimination of the Regge piece cannot be used for a low energy sum rule, since there will appear the on-shell amplitude at $\nu = q \cdot k = 0$, which is not easily known.

We now write down the final form of our sum rules at $k^2 = t = 0$. The ones of low energy type are

$$(\text{I, II}): \frac{4}{\pi}\int_{m+m_\pi}^{\infty} \frac{dWW}{W^2 - m^2}\frac{g_r}{g_r(0)} Im(U_1^\pi - mU_6^\pi)^{(+,0)}(W, q^2 = 0) = g_r \frac{\kappa^{(v,s)}}{2m^2} \tag{7.9}$$

$$(\text{III}): \frac{4}{\pi}\int_{m+m_\pi}^{\infty} \frac{dWW}{W^2 - m^2}\frac{g_r}{g_r(0)} Im U_8^{\pi(-)}(W, q^2 = 0) = \frac{g_r}{m}\left(F_1^{v'}(0) - \frac{g_A'(0)}{g_A(0)}\right) \tag{7.10}$$

$$\frac{4}{\pi}\int_{m+m_\pi}^{\infty} \frac{dWW}{W^2 - m^2} Im\left[\frac{g_r}{g_r(0)}(U_1^\pi + 2U_4^\pi) - RP\right]^{(-)}(W, q^2 = 0) = \frac{f_\pi g_r}{mg_A(0)}(2mg_A(0)F_1^{v'}(0) - I'(0) - 2S_4') \tag{7.11}$$

while the superconvergence type sum rules may be written in the general form

$$(\text{IV–XI}):\frac{2}{f_\pi}\int_{m+m_\pi}^{\infty} dW\,W\,Im\left[U_i^{\pi(\pm,0)}(W, q^2 = m_\pi^2) - \lambda\frac{g_r}{g_r(0)}U_i^{\pi(\pm,0)}(W, q^2 = 0)\right] = g_r R_i \Delta + S_i'/f_\pi \quad (7.12)$$

Here RP denotes the Regge piece to be subtracted out before going to the fixed mass limit, Δ measures the discrepancy in the Goldberger-Treiman relation, $\Delta = 1 - \lambda$ where $\lambda = mg_A(0)/f_\pi g_r$. The pion pole is understood to have been eliminated from both sides of the sum rules (even if it does not cause divergence of the integral). The details of the superconvergence type sum rules are given in Table III. As $(\bar{U}_3 + \bar{U}_4)$ has no definite symmetry under $\nu \to -\nu$ with masses held fixed, we have chosen to write the sum rule for the combination $(2U_3 + U_1 + 2U_4)$ which is even and odd for isospin $(+, 0)$ and $(-)$ respectively.

Table III
Details of the superconvergence type sum rules. See Equation (7.12) in the text.

Sum rule	Invariant amplitude	R_i	Scaling function
IV	$U_1^{(-)}$	$1 + \kappa^v$	$S_1^{(-)}$
V	$U_6^{(-)}$	κ^v/m	$S_6^{(-)}$
VI	$U_7^{(+)}$	$\kappa^v/2m$	$S_7^{(+)}$
VII	$U_8^{(+)}$	0	$S_8^{(+)}$
VIII	$(2U_3 + U_1 + 2U_4)^{(+)}$	-1	$S_{314}^{(+)}$
IX	$U_7^{(0)}$	$\kappa^s/2m$	$S_7^{(0)}$
X	$U_8^{(0)}$	0	$S_8^{(0)}$
XI	$(2U_3 + U_1 + 2U_4)^{(0)}$	-1	$S_{314}^{(0)}$

As already mentioned, the degree of accuracy required to evaluate the Regge piece (RP) in sum rule (7.11) is out of question at present. We therefore leave it out in our numerical evaluation in the next section. It will be recalled that sum rules (I), (II) and (III) are exactly the ones obtained originally [5, 6] on the assumption of unsubtracted dispersion relations for the relevant amplitudes. These sum rules thus receive no correction from the behaviour of current commutators on the light cone. The low energy sum rule (5.9) for $U_2^{(+,0)}$ has dropped out from our list as the entire amplitude U_2 (as also its scaling function) is zero at $q^2 = k^2 = t = 0$.[5])

8. Numerical evaluation

The off-shell invariant amplitudes are evaluated using Adler's model [18]: pion mass is set equal to zero everywhere in the kinematic factors and the multipoles

[5]) This follows by relating U_2 to the isobaric frame amplitudes. See, for example, Reference [13].

corresponding to mass zero external pion are related to the physical multipole by

$$\mathcal{M}_{l\pm}(q^2 = 0) = \frac{g_r(0)}{g_r} \mathcal{M}_{l\pm}(q^2 = m_\pi^2) \left(\frac{W^2 - m^2}{2Wq}\right)^l,$$

where $\mathcal{M}^l_{\pm}$ stand for any of the magnetic, electric or longitudinal multipoles and q is the magnitude of πN centre of mass 3 momentum.

The longitudinal multipoles[6]) are evaluated by extending their known proportionalities to the electric multipoles at zero 3 momentum of photon [13] to non-zero 3 momenta as well. As pointed out in Reference [6] this, of course, is a source of much uncertainty in the evaluation of those sum rules where the longitudinal multipoles are the major contributors. As can be seen from Table IV this is the case with sum rules III, VII, VIII, X, XI.

The values of the physical constants used in the numerical evaluation are $m = 6.72\, m_\pi$, $g_A(0) = 1.25$, $f_\pi = 0.665 m_\pi$ and $g_r = 13.40$, giving [20]

$$\Delta = 1 - \frac{m g_A(0)}{f_\pi g_r} = 0.058 \pm 0.013.$$

$g'_A(0)$ is evaluated by using a dipole from factor for $g_A(t)$ with $m_A = 1$ GeV [21].

The evaluations are shown in Table IV. For completeness, we have also evaluated the three old low energy type sum rules (I, II and III) with the new multipole fits to the photoproduction data. The multipole analysis of Crawford et al. [22] has been used to evaluate the integrals appearing in the sum rules, restricting the *cm* energy up to 1.8 GeV and the orbital angular momentum of the multipoles up to 3. These numbers are shown without bracket in the table. To see how different analyses compare with one another in the evaluation of the integals, we have also evaluated these with the multipole analysis of Moorhouse et al. [23].[7]) These are shown in brackets in the table. The discrepancy between the two evaluations is, perhaps, a good measure of the error involved in the current multipole analyses.

The sum rules are in fair agreement with our estimate for the magnitude of the retarded scaling functions, given in Appendix (A19, 20). Note that the scaling functions for isospin $(-)$ amplitudes do not involve the quark mass m_q explicitly. The estimate for isospin $(+, 0)$ scaling functions are written for $m_q = 10\,\text{MeV}$. Thus even if our estimate for the scaling functions are too large by a factor of 10, the value for m_q cannot, on the basis of our crude evaluation, exceed $100\,\text{MeV}$. This bound, of course, satisfies other determinations of m_q [24, 25, 26, 27].

9. Conclusion

Our work describes the derivation of light cone sum rules for the current commutator involving electromagnetic and divergence of axial vector currents and the limiting cases of these for which both current masses remain fixed. We then find it

[6]) Devenish et al. [19] have performed a multipole analysis of pion electroproduction. However, their parametrization for the longitudinal multipoles do not respect the above mentioned proportionalities with electric multipoles at zero 3 momentum of photon.

[7]) The tables of partial waves in Reference [23] does not extend up to threshold. Evaluation over the missing region is carried out with the analysis of Crawford et al. [22].

Table IV
Evaluation of sum rules. Some of the sum rules are multiplied by appropriate powers of m, the nucleon mass, (as shown in the first row) so that all quantities in this table are expressed in units of m_π^{-2}.

contribution from \ sum rule		I	II	III × m	IV/m^2	V/m	VI/m	VII/m	VIII/m^2	IX/m^2	X/m	XI/m^2
multipoles	$M_{1+}^{3/2}$	0.421 (0.441)	0	0.092 (0.096)	0.065 (0.068)	0.069 (0.073)	0.027 (0.028)	−0.005	−0.001 (−0.002)	0	0	0
	remaining magnetic	−0.090 (−0.060)	−0.036 (−0.035)	0.019 (0.015)	−0.016 (−0.010)	−0.015 (−0.009)	−0.002 (−0.001)	−0.002 (−0.001)	0.008 (0.005)	0.0	−0.001	0.004
	electric	0.135 (0.130)	0.020 (−0.003)	0.0 (0.021)	0.0	−0.003	0.005 (0.010)	0.023 (0.013)	0.001 (0.005)	0.001 (0.003)	0.0 (−0.002)	0.001 (0.002)
	longitudinal	0	0	−0.069 (−0.080)	0	0	0	−0.034 (−0.022)	−0.001 (0.007)	0	0.0 (0.001)	0.005 (0.002)
left hand side		0.466 (0.511)	−0.016 (−0.038)	0.042 (0.052)	0.049 (0.058)	0.051 (0.061)	0.030 (0.037)	−0.018 (−0.015)	0.007 (0.015)	0.001 (0.003)	−0.001 (−0.002)	0.010 (0.008)
right hand side		0.549	−0.018	−0.060	$\dfrac{0.080 + S_1^{(-)\prime}}{(m^2 f_\pi)}$	$\dfrac{0.063 + S_6^{(-)\prime}}{(m f_\pi)}$	$\dfrac{0.032 + S_7^{(+)\prime}}{(m f_\pi)}$	$\dfrac{S_8^{(+)\prime}}{(m f_\pi)}$	$\dfrac{-0.017 + S_{314}^{(+)\prime}}{(m^2 f_\pi)}$	$\dfrac{-0.001 + S_7^{(0)\prime}}{(m f_\pi)}$	$\dfrac{S_8^{(0)\prime}}{(m f_\pi)}$	$\dfrac{-0.017 + S_{314}^{(0)\prime}}{(m^2 f_\pi)}$

necessary, for practical evaluation, to eliminate the Regge piece from the superconvergence type sum rules, thus obtaining the final set of sum rules given by equations (7.9–7.12).

We wish to summarize the major assumptions entering the derivation and evaluation of the sum rules. The derivation proceeds within the framework of Reference [4] which is based on causality and quark light cone algebra of currents. At the level of practical evaluation, we have to take resort to further assumptions. The current amplitudes involved in the sum rules are converted into zero mass pion electroproduction amplitudes with the PCAC choice for the pion field to effect the off-shell continuation. We employ Adler's model [18] to relate the zero mass pion amplitude to the physical amplitude. It must, however, be pointed out that this model plays a much more crucial role in our sum rules than, for example, in the well known sum rules for $g_A(0)$; whereas in the latter the off-shell character may be considered as introducing correction to the on-shell amplitude, the integrals in our (superconvergence type) sum rules derive their entire contributions from this correction only. However, if we do not accept the excellent agreement between Adler's evaluation of $g_A(0)$ [18] and its present experimental value as a mere chance, we must conclude that the model is accurate enough.

By far the most difficult task in subjecting our sum rules to an accurate experimental verification is to estimate the retarded scaling functions. There occurs too many form factors $\tilde{A}_i(\xi)$ in the decomposition of the matrix element for the quark bilocal about which we know too little. The Regge subtraction rendering the retarded scaling functions finite in the fixed mass limit introduces additional uncertainty in the magnitude of these quantities. In the Appendix, we make a plausible guess for the form of $\tilde{A}_i(\xi)$ based on their interpretation as subhadronic wave functions of quarks within the nucleon [24].

If should, of course, be possible to calculate the scaling functions from a sufficiently detailed dynamical model of quark interactions within the nucleon. Our sum rules may then prove to be a useful tool for checking in a phenomenological way such dynamical models or getting values of unknown parameters involved in the model. We have, however, not addressed ourselves to this difficult problem in the present work. We have thus only been able to set a rough upper bound of 100 MeV for m_q.

We next wish to compare briefly the present work with the extensive work of Furlan et al. [25, 26] based on the equal time commutator of electromagnetic and divergence of axial vector current. These authors saturate the nucleon matrix element of the equal time commutator in the Breit frame. This special choice of the reference frame has the merit of being able to make use of the usual crossing relations in spite of the fact that the dispersion integral runs over a parabola in (q^2, ν) plane. Furthermore, it allows the choice of a threshold kinematic configuration in which all contributions other than those of s waves are highly suppressed in the dispersion integral. PCAC hypothesis is used to relate off-shell amplitudes to physical ones and variation of pion mass in the amplitudes is ignored in the correction terms. In contrast, our sum rules derive no benefit from any special choice of reference frame. We cast our sum rules in the fixed mass limit before using PCAC hypothesis. A model [18] is then invoked to relate the zero mass amplitudes to the physical ones. One, however, faces large uncertainties, as already discussed, in evaluating the retarded scaling functions in contrast to simple form factors in the case of equal time commutator.

Finally we point out that the sum rules written down in this paper remain valid in the framework of asymptotically free gauge theories. However, the evaluation of scaling functions in the fixed mass limit satisfying the constraints of such theories is not a straightforward task. This point is now under investigation.

Acknowledgements

The author is very grateful to Professor H. Leutwyler for suggesting the problem, for valuable help and advice during innumerable clarifying discussions and for much encouragement. He also suggested improvements on the manuscript. The author is thankful to Professor P. Minkowski for very helpful discussions. Special thanks are due to Professor R. L. Crawford and Dr. N. H. Parsons for sending him the photoproduction multipole analysis carried out by his group. The author also thanks Professor M. Ida, Dr. J. Gasser, Dr. T. H. Go and Dr. R. L. Heimann for useful discussions.

Appendix

Here we first collect our information on the absorptive scaling functions in the form of sum rules obtainable from general considerations. These sum rules then form the basis of our estimate for the retarded scaling functions.

The relation

$$e_\mu \bar{T}^\mu_{Bj}(\xi, \xi_-, p_2, p_1) = -e_\mu \bar{T}^{\mu*}_{Bj}(-\xi, -\xi_-, p_1, p_2) \tag{A1}$$

yields the following symmetry relations for $\tilde{A}_i$,

$$\tilde{A}_i^{(\pm,0)}(\xi, \xi_-, t) = (\pm, +)\lambda_i \tilde{A}_i^{(\pm,0)}(-\xi, -\xi_-, t) \tag{A2}$$

where $\lambda_i = +1$ for $i = 2, 3, 4, 6, 13, 14, 16$ and $\lambda_i = -1$ for $i = 1, 5, 10, 11, 12, 15$, together with

$$\tilde{\mathscr{A}}_i^{(\pm,0)}(\xi, \xi_-, t) = (\pm, +)\mu_i \tilde{\mathscr{A}}_i^{(\pm,0)}(-\xi, -\xi_-, t) \tag{A2$'$}$$

where $\tilde{\mathscr{A}}_1 = \tilde{A}_7 + \tilde{A}_{13}, \tilde{\mathscr{A}}_2 = \tilde{A}_8 + \tilde{A}_{12}, \tilde{\mathscr{A}}_3 = \tilde{A}_9 + 2\tilde{A}_{10}$; $\mu_1 = -1$, $\mu_2 = \mu_3 = +1$.

Fourier inversion of (3.5),

$$K(P\cdot x)\tilde{A}_i^{(\pm,0)}(P\cdot x) = \int_{-1}^{+1} d\xi\, e^{-i\xi P\cdot x}\tilde{A}_i^{(\pm,0)}(\xi, \xi_-, t) \tag{A3}$$

and the finiteness of $A_i(P\cdot x)$ at $P\cdot x = 0$ yields

$$\int_{-1}^{+1} d\xi \tilde{A}_i^{(\pm,0)}(\xi, \xi_-, t) = 0, \quad i = 2, (4), (5), (6)$$

and

$$\int_{-1}^{+1} d\xi\xi \tilde{A}_{(6)}^{(\pm,0)}(\xi, \xi_-, t) = 0. \tag{A4}$$

At $\xi_- = 0$, some of these relations are identically satisfied due to the symmetry relations (A2).

From now on we specialize to $\xi_- = 0$ (but $t \neq 0$) for simplicity and later use. Inserting (A3) into (3.3) with x on the light cone ($x = \lambda n$, $n^2 = 0$) we get

$$C_{\mu\nu}^{(\pm,0)}(x) = \int_{-1}^{+1} d\xi\, e^{-i\xi P\cdot x}\bar{u}(p_2)\gamma_5 \frac{a^{(\pm,0)}}{2}\Bigg[g_{\mu\nu}\tilde{A}_1^{(\pm,0)} + g_{\mu\nu}\frac{\not{x}}{P\cdot x}\tilde{A}_2^{(\pm,0)} + a_{\mu\nu}\tilde{A}_{(3)}^{(\pm,0)} + \frac{(b_\mu x_\nu - b_\nu x_\mu)}{P\cdot x}\tilde{A}_{(4)}^{(\pm,0)} + a_{\mu\nu}\frac{\not{x}}{P\cdot x}\tilde{A}_{(5)}^{(\pm,0)} + \frac{(b_\mu x_\nu - b_\nu x_\mu)\not{x}}{(P\cdot x)^2}\tilde{A}_{(6)}^{(\pm,0)}\Bigg]u(p_1). \tag{A5}$$

To proceed further, we have to treat isospin index $(-)$ and $(+, 0)$ separately. Consider the local limit of (A5) for isospin $(-)$ case first. The left hand side of (A5) yields

$$\frac{g_{\mu\nu}}{2m_q} H(t)\bar{u}(p_2)a^{(-)}\gamma_5 u(p_1)$$

and we get a new sum rule

$$\int_{-1}^{+1} d\xi \tilde{A}^{(-)}(\xi) = \frac{H(t)}{m_q} \tag{A6}$$

Further sum rules are obtained by differentiating both sides of (A5) with respect to x (after shifting the x dependence by $\pm x/2$), making use of free Dirac equation for the quark fields and finally going to the local limit. After removing those integrals which are zero by symmetry, we get

$$\int_{-1}^{+1} d\xi\xi[\tilde{A}_3^{(-)} - P^2\tilde{A}_4^{(-)} + 3(\tilde{A}_8^{(-)} + \tilde{A}_{12}^{(-)}) - \tfrac{1}{2}\tilde{A}_{13}^{(-)}] = 2m_q h_A(t) - \frac{H(t)}{m_q} \tag{A7}$$

$$\int_{-1}^{+1} d\xi\xi[-\tilde{A}_2^{(-)} - 2m\tilde{A}_3^{(-)} + P^2\tilde{A}_6^{(-)} - 3(\tilde{A}_9^{(-)} + 2\tilde{A}_{10}^{(-)}) + m\tilde{A}_{13}^{(-)}] = 2m_q g_A(t) \tag{A8}$$

In accordance with our discussion in section 6, we eliminate the pion pole contributions from these equations. As may be seen from (6.1), (6.7) and (3.3) only $A_1^{(-)}$ and $A_8^{(-)}$ get contribution from the pion pole. Denoting this elimination by a prime, (A6) and (A7) are replaced by

$$\int_{-1}^{+1} d\xi \tilde{A}_1^{(-)\prime}(\xi) = \frac{I(t)}{m_q} \tag{A9}$$

$$\int_{-1}^{+1} d\xi\xi[\tilde{A}_3^{(-)} - P^2\tilde{A}_4^{(-)} + 3(\tilde{A}_8^{(-)\prime} + \tilde{A}_{12}^{(-)}) - \tfrac{1}{2}\tilde{A}_{13}^{(-)}] = -\frac{I(t)}{2m_q} \tag{A10}$$

where we have ignored quantities proportional to m_q on the right of (A10). We recall that $I(t)$ is assumed to be proportional to m_π^2, so that the right hand sides of (A9) and (A10) are finite in the chiral limit.

Sum rules for isospin symmetric cases may be obtained in a similar way. Local limit of (A5) results in the sum rules (the antisymmetric kinematic covariants remaining after use of (A4) are linearly independent),

$$\int_{-1}^{+1} d\xi \tilde{A}_i^{(\pm,0)}(\xi, t) = G_j^{(\pm,0)}(t), \quad i = 3, 4, 6; j = i - 2$$

$$\int_{-1}^{+1} d\xi \tilde{A}_i^{(\pm,0)}(\xi, t) = 0, \quad i = 1, 5 \tag{A11}$$

where G_i's are form factors in

$$-2i\langle p_2 \mid \bar{q}(0)\gamma_5\bar{\sigma}_{\mu\nu}\Lambda^{(+,0)}q(0) \mid p_1\rangle$$
$$= \bar{u}(p_2)\gamma_5(a^{(\pm,0)}/2)[\sigma_{\mu\nu}G_1^{(+,0)}(t) + (P_\mu\Delta_\nu - P_\nu\Delta_\mu)G_2^{(+,0)}(t) + (P_\mu\gamma_\nu - P_\nu\gamma_\mu)G_3^{(+,0)}(t)]u(p_1) \tag{A12}$$

(time reversal does not allow a term with $\gamma_5(\Delta_\mu\gamma_\nu - \Delta_\nu\gamma_\mu)$). A rough estimate of $G_1^{(+,0)}(t = 0)$ is obtained by considering the static limit of (A12) and invoking static SU(6) symmetry among the nonet of axial vector currents,

$$G_1^{(0)}(t = 0) = 5/9, \quad G_1^{(+)}(t = 0) = 5/6. \tag{A13}$$

A sum rule analogous to (A7) and (A8) for isospin (+) case is

$$\int_{-1}^{+1} d\xi\xi[\tilde{A}_1^{(+,0)} + 3(\tilde{A}_7 + \tilde{A}_{13})^{(+,0)} - 2(m\tilde{A}_{11}^{(+,0)} + \tilde{A}_{12}^{(+,0)})] = -2G_1^{(+,0)}(t) - (t/2)G_2^{(+,0)}(t) \tag{A14}$$

We have now written above all the sum rules satisfied by the absorptive part of the scaling functions $\tilde{A}_i^{(\pm,0)}(\xi, t)$ which are exact within the framework of the quark model considered. Turning now to the retarded scaling functions $S_i^{(\pm,0)}(\xi, t)$ in the fixed mass limit ($\xi = \xi_- = 0$), our discussion in Section 7 shows that we have first to subtract from $S_i^{(\pm,0)}$ (or $A_i^{(\pm,0)}$), those contributions which are singular or finite in this limit. The subtracted scaling functions, denoted by $\tilde{A}_i'$, still satisfy the symmetry relations but the sum rules are now to be considered as rough inequalities. The subtracted retarded scaling functions in the fixed mass limit take the general form

$$S_i'^{(\pm,0)}(\xi = \xi_- = 0, t) = m_q \int_0^{+1} \frac{d\xi'}{\xi'} \sum \tilde{A}_i'^{(\pm,0)}(\xi', \xi_- = 0, t) \tag{A15}$$

where $\sum \tilde{A}_i'^{(\pm,0)}$ denote the combination of $\tilde{A}_i$ appearing in (3.7). Of course, only those $\tilde{A}_i^{(\pm,0)}$ which are odd in ξ can appear in the retarded scaling functions. It is easy to check that the non-vanishing $S_i^{(\pm,0)}$ are in conformity with the fixed pole behaviour of the amplitudes listed in Table II.

For a rough estimate of the retarded scaling functions we assume odd $\tilde{A}_i^{(\pm,0)}$ to have the form [15]

$$\tilde{A}_i^{(\pm,0)}(\xi, \xi_- = 0, t = 0) = a_i^{(\pm,0)}\xi(1 - \xi^2) \tag{A16}$$

where $a_i^{(\pm,0)}$ are constants. Then the sum rules (A10) and (A14) yield algebraic relations amongst $a_i^{(\pm,0)}$:

$$a_3^{(-)} - \tfrac{1}{2}a_{13}^{(-)} - m^2 a_4^{(-)} + 3(a_8^{(-)} + a_{12}^{(-)}) = -(15/8)(I(0)/m_q) \simeq 2(m_\pi/m_q) \tag{A17}$$

$$a_1^{(+,0)} + 3(a_7^{(+,0)} + a_{13}^{(+,0)} - 2(ma_{11}^{(+,0)} + a_{12}^{(+,0)}) = -(15/2)G_1^{(+,0)}(0) \tag{A18}$$

Clearly we have too few equations to solve for $a_i^{(\pm,0)}$. Our last crude assumption consists in assuming that each of the terms in (A17) are[8]) $\sim m_\pi/m_q$ and those in (A18) are ~ 5. Thus the scaling functions appearing in the sum rules are estimated as

$$|S_1'^{(-)}/m^2| \simeq |S_6'^{(-)}/m| \simeq m_\pi/m^2 \simeq 0.02\, m_\pi^{-1} \tag{A19}$$

$$|S_7'^{(+,0)}/m| \simeq |S_8'^{(+,0)}/m| \simeq S_{314}'^{(+,0)}/m^2| \simeq (4m_q/m^2) \simeq 0.005\, m_\pi^{-1}(m_q = 10 MeV). \tag{A20}$$

Note that the estimates for isospin $(-)$ scaling amplitudes do not depend directly on m_q and is obtained from the observed discrepancy in the Goldberger–Treiman relation.

REFERENCES

[1] H. Leutwyler and J. Stern, Nucl. Phys., *B20*, 77 (1970).
[2] R. Jackiw, R. Van Royen and G. West, Phys. Rev. *D2*, 2473 (1970); D. Dicus, R. Jackiw and V. Teplitz, Phys. Rev., *D4*, 1733 (1971).
[3] H. Fritzsch and M. Gell-Mann, Tracts in mathematics and natural sciences, vol. 2 (Gordon and Breach, 1971).
[4] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Light cone algebra, fixed mass sum rules and PCAC (unpublished).
[5] S. Fubini, G. Furlan and C. Rosetti, Nuovo Cimento, *40A*, 1171 (1965); G. Furlan, R. Jengo and E. Remiddi, Nuovo Cimento, *44A*, 427 (1966); Riazuddin and B. W. Lee, Phys. Rev., *146B*, 1202 (1966).
[6] S. L. Adler and F. J. Gilman, Phys. Rev., *152*, 1460 (1966).
[7] R. Dolen, D. Horn and C. Schmid, Phys. Rev., *166*, 1768 (1968); K. Igi and S. Matsuda, Phys. Rev. Lett., *18*, 625 (1967); A. Logunov, L. D. Soloviev and A. N. Tavkhelidze, Phys. Lett., *B24*, 181 (1967).
[8] R. Odorico, Nucl. Phys., *B101*, 480 (1975).
[9] Original reference may be found in H. D. Politzer, Phys. Rep., *14C*, 130 (1974).
[10] H. Fritzsch and P. Minkowski, Nuovo Cimento, *30A*, 393 (1975).
[11] J. S. Ball, Phys. Rev., *124*, 2014 (1961).
[12] M. Nauenberg, Phys. Lett., *22*, 201 (1966).
[13] S. L. Adler, Ann. Phys., *50*, 189 (1968).
[14] H. Leutwyler, "Currents on the light cone" in Proceedings of Summer Institute in Theoretical Physics in Kaiserslautern, 1972.
[15] H. Leutwyler, "Light Cone Physics and PCAC" in Proceedings of the Adriatic Summer Meetings on Particle Physics, Rovinj (Yugoslavia), 1973.
[16] J. S. Ball and M. Jacob, Nuovo Cimento, *54*, 620 (1968); N. Dombey, Nuovo Cimento, *31*, 1025 (1964).
[17] M. Gell-Mann and M. Lévy, Nuovo Cimento, *16*, 705 (1960); Y. Nambu, Phys. Rev. Lett., *4*, 380 (1960).
[18] S. L. Adler, Phys. Rev., *140B*, 736 (1965); see also Reference [6].
[19] R. C. E. Devenish and D. H. Lyth, Nucl. Phys., *B93*, 109 (1975).

---

[8]) The sum rules for $a_i^{(-)}$ obtained from (A8) has a much smaller right hand side, which, for consistency with (A17) must be due to large cancellations among different terms. We therefore ignore this sum rule in arriving at our rough estimates.

[20] H. F. JONES and M. D. SCADRON, Phys. Rev., *11D*, 174 (1975).
[21] A. B. CLEGG in Proceedings of the XI International Symposium on Electron and Photon Interactions at High Energies (Eds. H. ROLLNIK and W. PFEIL).
[22] Private communication. See also I. M. BARBOUR, R. L. CRAWFORD and N. H. PARSONS, preprint (University of Glasgow).
[23] R. G. MOORHOUSE, H. OBERLACK and A. H. ROSENFELD, Phys. Rev., *D9*, 1 (1974).
[24] H. LEUTWYLER, Nucl. Phys., *B76*, 413 (1974).
[25] G. FURLAN, N. PAVER and C. VERZEGNASSI, Nuovo Cimento, *32A*, 75 (1976); *20A*, 259 (1974).
[26] G. FURLAN, N. PAVER and C. VERZEGNASSI, Nuovo Cimento, *62A*, 519 (1969).
[27] M. TESTA, Phys. Lett., *56B*, 53 (1975).