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Real space dynamic renormalization group for an Ising spin glass

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Abstract. A kinetic version of a spin-glass model is studied in the framework of a Migdal like real space dynamic renormalization group transformation. Static and dynamic exponents are computed for the Ising, tricritical and spin-glass fixed points in two and three dimensions.

1. Introduction

Much effort has been recently expended in attempting to explain the behaviour of spin-glasses. Several microscopic models have been proposed to describe the qualitative static features experimentally observed at the spin-glass transition; namely a cusped susceptibility and a smoothly rounded specific heat [1]. All those models consist of a system of spins on a lattice interacting via a random coupling constant J_{ij}. An important characterization of a given model is the probability distribution $P(J_{ij})$ for the coupling constants J_{ij}. It has been recognized that in order to simulate a spin-glass, a model should have at least two properties: frustation [2] and disorder. All the models studied until now can be classified into two classes: (i) the Edwards-Anderson type [3] for which $P(J_{ij})$ is a gaussian with mean J_0 (often chosen equal to zero; (ii) the 'frustated type' for which J_{ij} can take the values $\pm J_0$ with probabilities p and $q = 1 - p$ (double peaked distribution). Despite the enormous number of papers devoted to this problem, there are still no clear cut answers to questions as what is the exact nature of the spin-glass phase? (order parameter ?), what is the lower critical dimensionality d_c? or how important the details of the probability distribution $P(J_{ij})$ are? [4]

For dynamics, the situation is even less clear. Below T_c, remanent effects or slow non exponential decay are observed. For T approaching T_c from above, the best information available is from Monte-Carlo simulations. Note however that this approach is particularly difficult for spin-glasses due to important metastability effects [5]. For two-dimensional ($d = 2$) models with gaussian distribution, the decay towards equilibrium seems to be a slow non-exponential one [5], while for

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$d=2$ and 3 and a double peaked distribution, one finds an usual exponential decay [6, 7].

Theoretically, the study of dynamics is restricted to mean field type calculations [8]. However, mean field results are only valid at dimensions $d > d_0 = 6$ [9] and thus, meaningless at $d=2$ or 3.

In view of the recent developments in real space dynamic renormalization group, it is legitimated to study a kinetic version of a spin-glass. The purpose of the paper is to investigate an Ising spin-glass model by means of a Migdal type dynamic renormalization group transformation [10, 11] generalized to inhomogeneous couplings.

The paper is organized as follows: in Section 2, the statics is analyzed for $d=2$ and 3. The phase diagram exhibits paramagnetic, ferromagnetic and spin-glass phases. The spin-glass and paramagnetic phases meet along a second order phase boundary which terminates in two tricritical points (see Fig. 2). The exponents are calculated for each fixed point (see Tables 1 and 2). In Section 3 a kinetic version of the model is defined and solved for $d=2$ and 3 by a Migdal-type dynamic renormalization group. The dynamic exponents z and $\Delta = \nu z$ (ν being the usual correlation length exponent) are given in Table 3 for $d=2$ and Table 4 for $d=3$. Finally, the results are discussed in Section 4.

2. Statics

The model is defined by the reduced Hamiltonian:

$$H = \sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j = \sum_i E_j\sigma_j \tag{1}$$

$$\sigma_i = \pm 1 \qquad J_{ij} = -K_{ij}/k_B T \tag{2}$$

where

$$E_j = \sum_i J_{ij}\sigma_i \tag{3}$$

The reduced coupling constants, J_{ij}, are supposed to be independent random variables with assigned probability distribution:

$$\begin{aligned} &P(J_{ij}) = p\delta(J_{ij} - J_0) + q\delta(J_{ij} + J_0) \\ &p+q+1 \qquad p, q \geqslant 0 \end{aligned} \tag{4}$$

The static renormalization group transformation consists in the following steps:

1. From the blocks of linear size a, we form new ones of size ba by moving the bonds inside the new blocks onto the sides (see Fig. 1). The scale factor b has to be chosen odd to preserve the ferro-antiferromagnetic symmetry and we choose the smallest possible value, $b=3$.
2. The remaining bonds $\mathbf{J}_i^\alpha$ are distributed according to:

$$P(\mathbf{J}_i^\alpha) = \sum_{k=0}^{b^{d-1}} \binom{b^{d-1}}{k} \delta\,(\mathbf{J}_i^\alpha - (b^{d-1} - 2k)J_0) \tag{5}$$

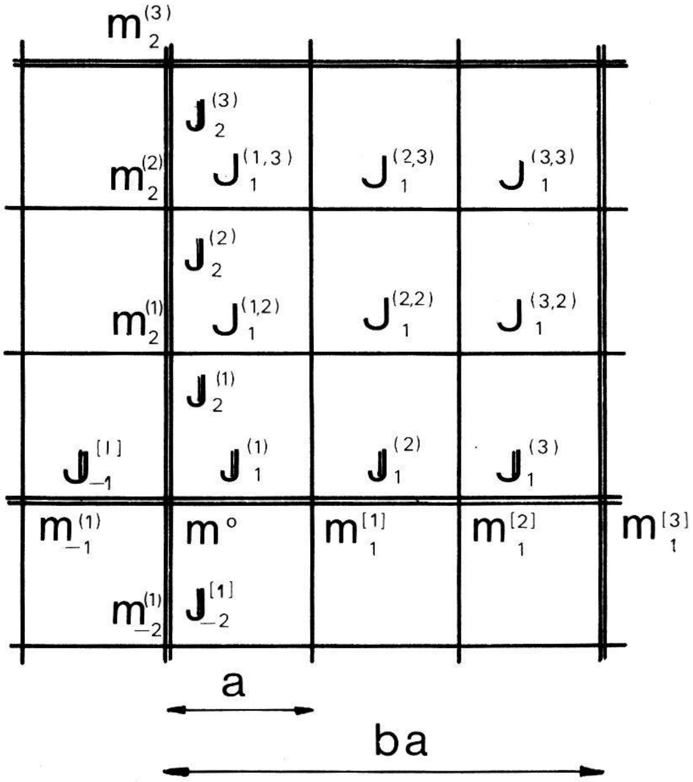

Figure 1
Illustration of the R.G. transformation. $J_i^{(\alpha,\beta)}$ are the original couplings, $\mathbf{J}_i^\alpha$ (thick lines, $\alpha = 1, 2, 3$) the couplings after bond-moving. Finally the intermediary spins are decimated (e.g. $m_1^{(1)}, m_1^{(2)}, \ldots$)

where d is the dimensionality of the lattice. We now proceed to approximate this form by the following double peaked one:

$$\bar{p}(\mathbf{J}_i^\alpha) = \bar{p}\delta(\mathbf{J}_i^\alpha - \bar{J}_{0n}) + \bar{q}\delta(\mathbf{J}_i^\alpha + \bar{J}_{0n}) \tag{6}$$

where

$$\bar{p} = \sum_{k=0}^{[b^{d-1/2}]} \binom{[b^{d-1/2}]}{k} p^{[b^{d-1/2}-k]} q^k; \qquad \bar{q} = 1 - \bar{p} \tag{7}$$

and

$$(\bar{J}_{0n})^n = c_n J_0^n, \qquad n = 1, 2, \ldots \tag{8}$$

with

$$c_n = \sum_{k=0}^{b^{d-1}} |b^{d-1} - 2k|^n \binom{b^{d-1}}{k} p^{(b^{d-1}-k)} q^k \tag{9}$$

This approximation assigns to $\bar{J}_{0n}$ all wieght in (4) and matches averages of J_0^n. For a detailed discussion of this point see the Ref. [12].

3. One dimensional decimation of the irrelevant spin variables on the sides of the new blocks. Note that our transformation does not suffer from the drawbacks pointed out by Kirkpatrick [14] concerning the standard Migdal renormalization group method and that it is different from the one of Ref. [12]. One hopes that at least the qualitative features of the problem are independent of n, which is indeed the case.

After completing the above steps the recussion relation for the inhomogeneous system reads:

$$\mathrm{th}\,(J_i') = \prod_{\alpha=1}^{3} \mathrm{th}\,\mathbf{J}_i^{\alpha} \tag{10}$$

We use this relation to renormalize the distribution function (4) according to:

$$\begin{aligned} P'(J_i') &= \int \prod_{\alpha=1}^{3} \{d\mathbf{J}_i^{\alpha}\bar{P}(\mathbf{J}_i^{\alpha})\}\, \delta\left\{J_i' - \mathrm{Arc\,th}\left(\prod_{\gamma=1}^{3} \mathrm{th}\,\mathbf{J}_i^{\gamma}\right)\right\} \\ &= p'\delta(J_i' - J_{0n}') + q'\,\delta(J_i' + J_{0n}') \end{aligned} \tag{11}$$

where

$$p' = \bar{p}^3 + 3\bar{p}\bar{q}^2, \qquad q' = 1 - p'$$

and

$$J_{0n}' = \mathrm{Arc\,th}\{\mathrm{th}^3(\bar{J}_{0n})\} = \mathrm{Arc\,th}\{\mathrm{th}^3\,(c_n^{1/n}J_0)\} \tag{12}$$

These are the required recursion relations leading to the different fixed points mentioned above. Standard arguments lead to the evaluation of ν and α. The exponent y_p is defined by:

$$b^{y_p} = \left.\frac{dp'}{dp}\right|_{p=p^*} \tag{13}$$

Table 1

Fixed points and critical exponents for $d = 2$. Results for $n = 1, 2, 3$ are shown. y_p is defined by (13) and α is evaluated using the relation $2 - \alpha = d\nu$.

	n	J_0^*	ν	α	p^*	y_p
Ising		0.241	1.36	−0.72	1	$-\infty$
Spin-glass	1	1.095	2.76	−3.52	1/2	$-\infty$
	2	0.740	2.12	−2.24		
	3	0.557	1.82	−1.64		
Tricritical	1	0.397	1.58	−1.16	0.135	0.769
	2	0.340	1.49	−0.98	0.865	
	3	0.309	1.45	−0.90		

Table 2

Fixed points and critical exponents for $d = 3$. Results for $n = 1, 2, 3$ are shown. y_p is defined by (13) and α is evaluated using the relation $2 - \alpha = d\nu$.

	n	J_0^*	ν	α	p^*	y_p
Ising		0.039	1.09	−1.27	1	$-\infty$
Spin-glass	1	0.347	1.506	−2.52	1/2	$-\infty$
	2	0.240	1.358	−2.07		
	3	0.186	1.280	−1.84		
Tricritical	1	0.194	1.293	−1.88	0.343	0.775
	2	0.145	1.227	−1.68	0.657	
	3	0.119	1.196	−1.59		

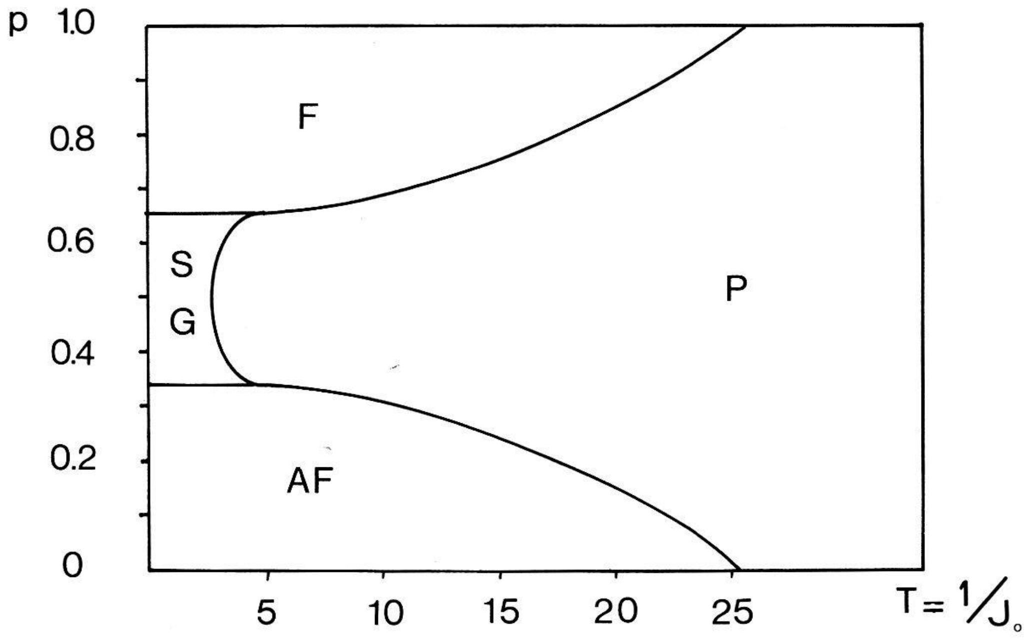

Figure 2
Phase diagram of the spin-glass model for $d = 2$. $1/J_0$ measures the temperature and p is the fraction of the ferromagnetic couplings. The four phases are the paramagnetic (P), the ferromagnetic (F), the antiferromagnetic (AF) and the spin-glass (SG). Fixed points are indicated as Ising (I), spin-glass (SG) and tricritical (T).

The results for $n = 1, 2, 3$, $d = 2$ are listed in Table 1, for $d = 3$ in Table 2 and a qualitative phase diagram is drawn in Fig. 2.

3. Dynamics

The kinetic model is defined by the following master equation for $\pi\{(\sigma); t\}$, the probability density that a state $\{\sigma\}$ is realized at time t:

$$\partial_t \pi\{(\sigma),t\} = -\sum_j \omega(\sigma_j)\pi\{(\cdots \sigma_i, \sigma_j \cdots), t\}$$

$$+\sum_j \omega(-\sigma_j)\pi\{(\cdots \sigma_{ij} - \sigma_j \cdots), t\} \tag{14}$$

Detailed balance is fulfilled by choosing:

$$\omega(\sigma_j) = \frac{1}{2\tau}[1 - \sigma_j \text{ th } E_j] \tag{15}$$

where τ, the relaxation time of a single spin interacting with the heat bath, is chosen to be homogeneous. The equation of motion for the one-point function $m_0(t) = \langle \sigma_0(t) \rangle$, reads:

$$\tau\, \partial_t m_0(t) = -m_0(t) + \langle \text{th } E_0 \rangle \tag{16}$$

Going to Laplace transform and expanding th E_0 in terms of the spin variables, one obtains (see Fig. 1 for the notation)

$$(1 + \tau s)m_0(s) = M_0 + \sum_{\alpha=-d}^{d} f_\alpha(\{J_\beta^{(1)}\})m_\alpha^{(1)}(s). \tag{17}$$

where $M_0 = m_0(t=0)$ and

$$f_\alpha(\{J_\beta^{(1)}\}) = f_\alpha(J_{-2}^{(1)}; J_{-1}^{(1)}; J_1^{(1)}; J_2^{(1)})$$
$$= \tfrac{1}{8}[\text{th}\,(J_\alpha^{(1)} + J_\beta^{(1)} + J_\gamma^{(1)} + J_\delta^{(1)}) + \text{th}\,(J_\alpha^{(1)} - J_\beta^{(1)} + J_\gamma^{(1)} + J_\delta^{(1)})$$
$$+ \text{th}\,(J_\alpha^{(1)} + J_\beta^{(1)} - J_\gamma^{(1)} + J_\delta^{(1)}) + \text{th}\,(J_\alpha^{(1)} + J_\beta^{(1)} + J_\gamma^{(1)} - J_\delta^{(1)})$$
$$+ \text{th}\,(J_\alpha^{(1)} + J_\beta^{(1)} - J_\gamma^{(1)} - J_\delta^{(1)}) + \text{th}\,(J_\alpha^{(1)} - J_\beta^{(1)} - J_\gamma^{(1)} + J_\delta^{(1)})$$
$$+ \text{th}\,(J_\alpha^{(1)} - J_\beta^{(1)} + J_\gamma^{(1)} - T_\delta^{(1)}) + \text{th}\,(J_\alpha^{(1)} - J_\beta^{(1)} - J_\gamma^{(1)} - J_\delta^{(1)})]$$

$$\alpha, \beta, \gamma, \delta = \circlearrowleft -2; -1; 1; 2$$

for $d = 2$ and similarly for $d = 3$, with $\alpha, \beta, \gamma, \delta, \epsilon, \nu = \circlearrowleft -3; -2; -1; 1; 2; 3$.

The higher order terms include two-point, three-point and four-point functions and are disregarded in first approximation [11].

The dynamic renormalization group transformation consists in three steps. The first two are the same as in statics, while the third one is a one dimensional dynamic decimation [10, 11]. Under this transformation inhomogeneities are introduced in the relaxation time τ. However the error involved by neglecting them can be estimated and it turns out that it is of the order of a few percent. This justifies an approximation consisting in forcing the relaxation time to remain homogenous.

Under the above transformation the equation (17) transforms as:

$$(1+\tau' s)m_0(s) = M_0' + \sum_{\alpha=-d}^{d} f_\alpha\{(J_\beta^{(1)'})\}m_\alpha^{(b)} + \text{higher orders} \tag{19}$$

where

$$f_\alpha\{(J_\beta^{(1)\prime})\} = \frac{f_\alpha\{(\mathbf{J}_\beta^{(1)})\}\psi^-(\mathbf{J}_\alpha^{(1)}; \mathbf{J}_\alpha^{(2)})\psi^-(\mathbf{J}_\alpha^{(2)}; \mathbf{J}_\alpha^{(3)})}{F(\mathbf{J}_\alpha^{(1)}; \mathbf{J}_\alpha^{(2)}; \mathbf{J}_\alpha^{(3)}) \cdot \left[1 - \sum_\gamma \frac{f_\gamma\{(\mathbf{J}_\sigma^{(1)})\}\psi^+(\mathbf{J}_\gamma^{(1)}; \mathbf{J}_\gamma^{(2)})}{E(\mathbf{J}_\gamma^{(1)}; \mathbf{J}_\gamma^{(2)}; \mathbf{J}_\gamma^{(3)})}\right]} \tag{20}$$

with

$$\psi^\pm(x; y) = \tfrac{1}{2}\{\text{th}\,(x+y) \pm \text{th}\,(x-y)\} \tag{21}$$

$$F(x; y; z) = 1 - \psi^-(x; y)\psi^-(y; z) \tag{22}$$

and

$$E(x; y; z) = 1 - \psi^+(x; y)\psi^+(y; z) \tag{23}$$

It is, in principle, possible to proceed exactly as in statics. However, it is easier to do an intermediary step by averaging equations (17) and (19) over all $\{J_\beta\}$'s with $\alpha \neq \beta$.

Again a slight inhomogeneity is introduced which turns out to be, numerically speaking, negligible. The partially averaged equations are then used to proceed as in statics. The resulting fixed point equation for p remains unchanged. (see equation (11)) while the fixed point equation for the coupling constant takes the form:

$$f_d(J_{0n}') = \frac{d f_d(\bar{J}_{0n})\psi_0^2(\bar{J}_{0n})}{4\left\{\frac{1}{d} E_0(\bar{J}_{0n}) - f_d(\bar{J}_{0n})\psi_0(\bar{J}_{0n})\right\}} \tag{24}$$

where

$$f_2(x) = \tfrac{1}{8}\{\text{th}\,(4x) + 2\,\text{th}\,(2x)\} \tag{25}$$

$$f_3(x) = \tfrac{1}{32}\{\text{th}\,(6x) + 4\,\text{th}\,(4x) + 5\,\text{th}\,(2x)\} \tag{26}$$

$$\psi_0(x) = \psi^+(x; x) \qquad E_0(x) = E(x; x; x) \tag{27}$$

and $\bar{J}_{0n}$ as given by (8).

The fixed points and the exponents ν and α are listed in Table 3 for $d = 2$ and in Table 4 for $d = 3$.

Table 3
Fixed points and critical exponents resulting from the dynamical R.G. transformation for $n = 1, 2, 3$. for $d = 2$.

	n	J_0^*	ν	α	p^*	y_p	z	Δ
Ising		0.187	0.80	0.40	1	$-\infty$	1.82	1.46
Spin-glass	1	0.478	0.51	0.98	$\frac{1}{2}$	$-\infty$	2.27	1.16
	2	0.404	0.56	0.88			2.21	1.24
	3	0.349	0.61	0.78			2.16	1.32
Tricritical	1	0.286	0.69	0.62	0.135	0.769	2.09	1.44
	2	0.258	0.73	0.54	0.865		2.05	1.50
	3	0.242	0.75	0.50			2.02	1.52

Table 4
Fixed points and critical exponents resulting from the dynamical R.G. transformation for $n = 1, 2, 3$. for $d = 3$.

	n	J_0^*	ν	α	p^*	y_p	z	Δ
Ising		0.035	1.006	−1.02	1	$-\infty$	1.41	1.42
Spin-glass	1	0.216	0.58	0.26	$\frac{1}{2}$	$-\infty$	2.20	1.28
	2	0.170	0.66	0.02			2.06	1.36
	3	0.134	0.85	−0.55			1.88	1.60
Tricritical	1	0.150	0.643	+0.07	0.343	0.755	2.01	1.30
	2	0.121	0.699	−0.10	0.657		1.98	1.38
	3	0.104	0.74	−0.22			1.92	1.42

By averaging the equations of motion [17, 19] one shows that the relaxation time characterizing the configurational average of the magnetization rescales as [11]

$$\frac{\tau'}{\tau} = 1 + \frac{2}{E_0}\left[1 + \frac{f_d(\bar{J}_{0n}^*)\psi_0(\bar{J}_{0n}^*)}{\frac{1}{d}E_0(\bar{J}_{0n}^*) - f_d(\bar{J}_{0n}^*)\psi_0(\bar{J}_{0n}^*)}\right] = b^z \tag{28}$$

z being the usual dynamic exponent. The values of z and of $\Delta = \nu z$ are listed in Table 3 for $d = 2$ and in Table 4 for $d = 3$.

4. Conclusions

Let us consider the statics first. For the two-dimensional case we find a spin-glass phase at finite temperature as Jayaprakash et al. [12] The phase diagram obtained is very reasonable, all the fixed points having the good stability (including the low temperature spin-glass fixed point $K^* = \infty$, $p = \frac{1}{2}$). The critical exponent α is smaller than -1, as it should in order to describe a rounded specific heat. However, several evidences are against the existence of a spin-glass at finite temperature for $d = 2$ [13]. The finite value of T_c obtained here may be due to the bond moving technique which leads to a over estimation of T_c [14]. However, the approximation should be good in the ground state ($T = 0$) and accordingly the prediction for p_t^*, delimiting the ferro-magnetic phase, should be good. Indeed $p_t = 0.135$ compares very well with other estimations by computer simultations [15]. For the three dimensional case, the above remarks concerning p_t^* and α are still valid. Moreover, for $d = 3$, it is very probable that a spin-glass phase exists at finite temperature and thus that the phase diagram obtained is meaningful.

Let us now consider the dynamics. A general feature of the dynamic renormalization group used here is the fact that the dynamic fixed points do not coincide with the static ones [16]. This inconsistency is intimately related to the truncation procedure consisting in neglecting the higher orders correlation functions in equation (17) [17]. However, despite this problem, the dynamic predictions for the two-dimensional pure Ising system are quite reasonable.

The critical exponents z and $\Delta = \nu z$ obtained for the spin-glasses are listed in Table 3 for $d = 2$ and Table 4 for $d = 3$. In both cases (and for all n) the values obtained do not differ very much from the pure case values. For the $2d$ case, the meaning of these results is questionable, due to the zero-critical temperature problem. However, for the $3d$ case, one obtains a usual exponential like critical slowing down with a dynamic exponent z somewhat higher than for the pure case. Note however that, due to the fact that we have to use a rescaling factor $b = 3$, the approximation for the spin-glass case is more drastic than the one used for the pure case. Indeed for the pure Ising case, one can improve the approximation by making an analytic continuation for $b = 1 + \delta b$ [16]. We found that such a continuation is not possible for the spin-glass problem in contradistinction with a recent paper by Forgacs et al. [18].

In conclusion, our calculation shows that the magnetization relaxes exponentially towards its equilibrium value. However, further investigations are needed to obtain a clear picture of the behaviour of the different random spin models and their relation to the spin-glasses.

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