

**Zeitschrift:** Helvetica Physica Acta  
**Band:** 62 (1989)  
**Heft:** 6-7

**Artikel:** Local electron scattering rates in noble metals  
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**DOI:** <https://doi.org/10.5169/seals-116065>

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## Local electron scattering rates in noble metals

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### INTRODUCTION

Since Pippard's first determination of the Fermi surface of copper in 1957 from anomalous skin effect experiments, followed by the first observations of the de Haas-van Alphen (dHvA) in a monovalent metal by Shoenberg, a great deal of theoretical and experimental works have been published and we have now a good knowledge of the Fermi surfaces of most elemental metals. The next step, in experiments such as dHvA, Azbel-Kaner cyclotron resonance (AKCR), surface Landau Level (SLL) <sup>1</sup> and radio frequency size effect (RFSE) <sup>2,3</sup>, is the study of electron scattering by phonons, electrons or impurities.

Ever since the seventies experimentalists were able to measure orbital averages of the temperature dependent electron scattering rates (SR) from electron-phonon (e-p) interaction (with RFSE or AKCR), and of the temperature independent scattering from electron-impurity interaction (with dHvA). Their variation over the Fermi surface was then obtained by inverting the data taken on many orbits to obtain local scattering data. The tilted field RFSE or SLL experiments give direct access to local scattering, however under stringent experimental conditions. The temperature dependent part of the SR was observed to be roughly proportional to  $T^3$  and therefore attributed totally to e-p in metals like Cu or Ag <sup>4,5</sup>. The agreement between different measurements was however not very good (fig 1), moreover effects from electron-electron interaction were not observed until recently, but in transition metals.

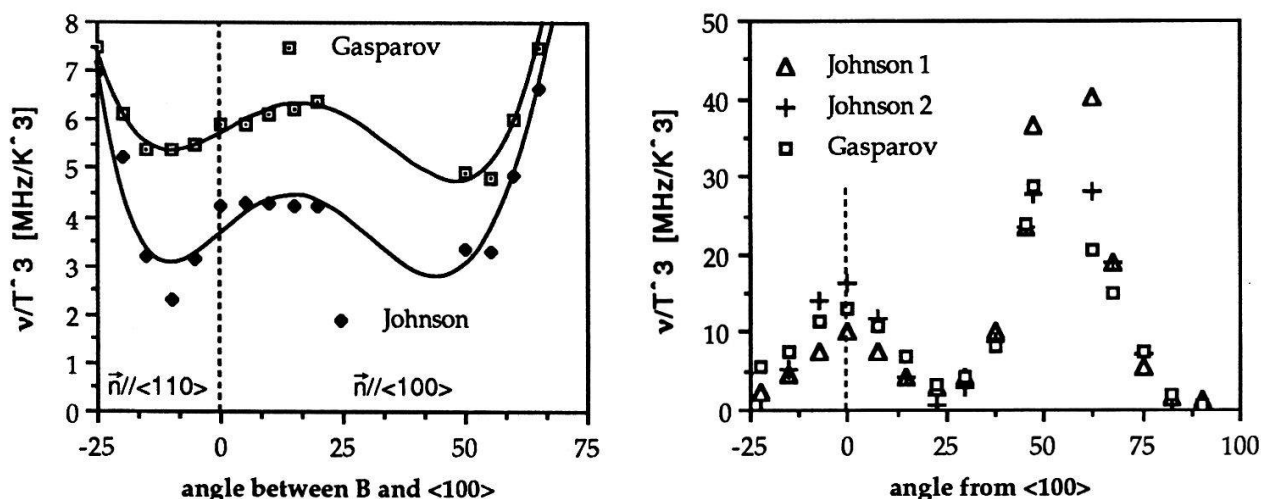


Figure 1: orbitally averaged scattering rates  $\bar{\nu}/T^3$  in Ag (left) and local scattering rates deduced by inversion

In Cd <sup>6</sup> we have found a diversity of behaviours of the SR, with temperature variations between  $T^2$  and  $T^3$  on first and second band orbits and even more rapid than  $T^5$  on the third band lens. The search for electron-electron (e-e) scattering led us to remeasure the SR in noble metals and to look for details in the scattering mechanism in order to resolve discrepancies between experimental data.

### RELAXATION TIME

The time rate of the distribution function  $\partial f/\partial t$  associated with a scattering mechanism may be written explicitly as the difference between terms involving the net scattering out of  $k$  and

into  $\mathbf{k}$ . Although the scattering probabilities for different mechanisms are additive, it does not necessarily follow that the corresponding collision integrals are also. The reason is that the deviation  $\psi(\mathbf{k})$  of the distribution function from the equilibrium depends implicitly on the scattering mechanism through the Boltzmann equation. This non-additivity of the collision integral is the source for deviations from Mattheissen's rule in the electrical resistivity.

On the contrary, in RFSE experiments,  $\psi(\mathbf{k})$  is localized in a small region of the Fermi surface (small compared to the scattering wave vector). In such cases the scattering occurs between the perturbed state  $\mathbf{k}$  and all other states  $\mathbf{k}'$  at equilibrium. Consequently one can neglect the contribution of the term containing  $\psi(\mathbf{k}')$  in the collision integral. It follows that i) the scattering may be characterized by a well defined scattering rate  $\text{SR } \nu(\mathbf{k}) = 1/\tau(\mathbf{k})$ :

$$\left( \frac{\partial f}{\partial t} \right)_{\text{coll}} = \frac{\partial f_0}{\partial \epsilon} \frac{\psi(\mathbf{k})}{\tau(\mathbf{k})} \quad (1)$$

ii) scattering rates associated with different scattering mechanisms are additive, making comparison with a theory easier.

## ELECTRON-PHONON INTERACTION

Assuming that the phonon distribution is in thermal equilibrium and that the electron distribution is displaced from equilibrium only at  $\mathbf{k}$ , the SR may be expressed by an integral over the phonon frequencies (Tomlinson and Carbotte<sup>7</sup>):

$$\frac{1}{\tau_{\text{ep}}(\mathbf{k})} = \frac{2\pi}{\hbar} \int_0^\infty d\omega \alpha^2 F(\omega) \{1 + f_0[\epsilon + \hbar\omega] - f_0[\epsilon - \hbar\omega] + 2n_0[\hbar\omega]\} \quad (2)$$

$n_0$  is the phonons distribution function and  $f_0$  the Fermi distribution function.  $\alpha^2 F(\omega)$  is the product of the e-p coupling function  $\alpha^2$  and the phonon density of states  $F(\omega)$

$$\alpha^2 F(\omega) = \frac{\Omega}{8\pi^3} \sum_{\lambda} \int_{\epsilon(\mathbf{k}')=\epsilon} \frac{dS'}{\hbar v'} |g_{\lambda}(\mathbf{k}, \mathbf{k}')|^2 \delta[\omega - \omega_{\lambda}(q)] \quad (3)$$

The strong variation with energy arises from the occupation factors. It follows generally from the properties of  $f_0$  and  $n_0$  that the SR is an even function of  $\epsilon$  with its minimum at the Fermi level. To calculate the SR, it is customary to use models for the dependence on energy and  $1/\tau_{\text{ep}}(\mathbf{k}, 0)$  may be evaluated (usually numerically) to determine the anisotropy, ie the dependence on  $\mathbf{k}$ . In the low-temperature limit, (Nowak<sup>8</sup>, Meador and Lawrence<sup>9</sup>) essentially all of the scattering is confined to a small region about  $\mathbf{k}$ . Therefore, the integration may be carried out in a plane tangent to the Fermi surface at  $\mathbf{k}$ . The surface element  $dS'$  is then  $\sim q dq$ . The matrix element squared being proportional to  $q$ , thus, for  $T \rightarrow 0$ , the e-p scattering obeys a  $T^3$  law.

## ELECTRON-ELECTRON INTERACTION

The main difficulty in the treatment of e-e scattering in metals<sup>10</sup> is the determination of the effective interaction potential  $V_{\text{ee}}(r)$  between two electrons. Thomas-Fermi and Bohm-Pines treatments of screening lead to a screened Coulomb interaction of the form

$$V_{ee}(r) = \frac{e^2}{4\pi\epsilon_0 r} \exp(-k_s r) \quad (4)$$

In the Bohm-Pines theory,  $k$  has half the size compared to Thomas-Fermi theory, therefore the magnitude of the interaction depends strongly on the model. In e-e interaction, the scattering angle are large, so e-e will be effective down to 0 K. The RFSE is sensitive to both normal and umklapp e-e scattering, in contrast to the electrical resistivity which is insensitive to normal e-e scattering in compensated metals. The observation of e-e is very difficult, because of the order of magnitude of the effect, and the recent review by Kaveh and Wyser<sup>11</sup> shows that the question is not completely settled. An e-e term is observed in  $\rho(T)$  with often a large scatter of the data; in  $v(k)$  an e-e term was observed only in transition metals until recently<sup>12 13</sup>.

## EXPERIMENTAL RESULTS

In the case of the thick samples used in the present investigation (1-2mm) the SR is related to the amplitude  $A(T)$  of the RFSE-resonance by the following expression

$$A(T) = C e^{-\bar{V}_0 t_c} e^{-\bar{V}(T) t_c} = A(0) e^{-\bar{V}(T) t_c} \quad (5)$$

where  $\bar{V}_0$  is due to physical and chemical impurities,  $\bar{V}(T)$  is the orbital averaged temperature dependent SR.  $C$  is a constant – for a given sample in a given coil – which enters into the amplitude  $A(0)$  at 0 K. The temperature dependent part of the SR  $V(T)$  is the sum of two contributions,  $v_{ee}(T) = \alpha T^2$  from e-e scattering and  $v_{ep}(T)$  from e-p scattering. We shall first assume that the latter follows the usual  $T^3$  - law,  $v_{ep}(T) = \beta T^3$ , so that  $V(T) = \alpha T^2 + \beta T^3$  which can be substituted in equation 5 to give:

$$\frac{1}{t_c T^2} \{\ln A(0) - \ln A(T)\} = \frac{V(T)}{T^2} = \alpha + \beta T \quad (6)$$

In fig. 2 it appears that  $v(T)/T^2$  does not show the linear dependence expected from equ. (6) and systematically decrease when the RF becomes smaller or/and the sample thinner. This means that some e-p scattering events must be non-effective when the phonon wave-vector  $q$  becomes too small. The change in effectiveness of scattering with temperature can be described quantitatively by a model first discussed by Myers et al's<sup>14</sup>. The e-p scattering rate of electrons at the Fermi level  $E_F$  can be expressed as

$$v_{ep}(E_F, T) = \beta(E_F) T^3 \left\{ \int_{T_\Sigma/T}^{\infty} \frac{x^2 e^x dx}{e^{2x}-1} + \left[ \frac{T}{T_\Sigma} \right]^2 \int_0^{T_\Sigma/T} \frac{x^4 e^x dx}{e^{2x}-1} \right\} \frac{4}{7\zeta(3)} \quad (7)$$

where  $x = \hbar q s / k_B T$  ( $s$  = sound velocity,  $k_B$  = Boltzmann constant).  $T_\Sigma$  is the temperature where the rms amplitude of the phonon wave vector  $q_{rms} = (k_B / 4.15 \hbar s) T$  is equal to  $\Delta k$ , the dimension of the region on the Fermi surface perturbed by the RF field.  $\Delta k$  is proportional to  $\delta/d$  or  $(\delta/d)^{1/2}$ . The term  $4/7\zeta(3) \cong 1/2.1$  is a normalizing constant which produces the correct value of  $\beta(E_F)$  when  $T_\Sigma = 0$ . Scattering will be completely effective if  $T > T_\Sigma$  and the second term in (7) will be negligible, leading to a  $T^3$  temperature dependence. In the other limit when  $T < T_\Sigma/10$ , the first term in (7) is negligible and  $v_{ep}(T)$  varies essentially like  $T^5$ . The parameter  $T_\Sigma$  determines the effectiveness at a given temperature and can be changed either by the thickness  $d$ , which means using different samples, or by  $\delta$ , which means working at various frequencies  $\omega$  with the same sample. The experimental data for the  $\langle 100 \rangle$ -orbit of Cu are plotted in

fig. 2 together with the solid curves calculated using equ. (7) together with an e-e term. Good agreement is indeed obtained at low temperature.

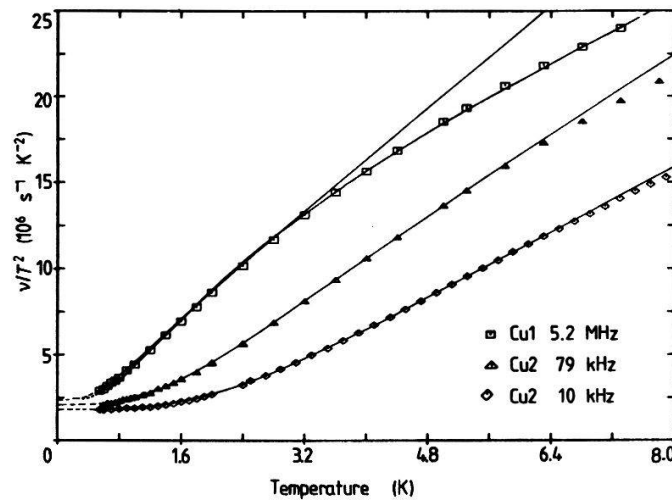


Figure 2:  $\bar{v}(T)/T^2$  for the  $\langle 100 \rangle$  orbit of Cu. Cu1 and Cu2 have thickness  $d$  of 1.89 and 1.03mm, respectively. The curves have been calculated using equ. 7 plus a term  $\alpha T^2$ .

Ineffectiveness depresses  $v(T)/T^2$  and if the measurements are not made at sufficiently low temperature, the extrapolation to 0K to determine  $\alpha$ , will wrongly appear to be zero or even negative. The data taken at low RF are best suited to show the existence of an  $\alpha T^2$ -term and to measure it, because e-p scattering is almost completely ineffective below 1.5K.

The values of  $\alpha$  from our RFSE-data depend slightly on the position of the orbit on the FS, being in Cu 25% and in Ag 50% smaller for the  $\langle 100 \rangle$ -orbit compared to the  $\langle 111 \rangle$ -orbit. e-e scattering is expected to be isotropic but the difference is small and larger variations over the FS have been reported in W from experiment and from theoretical calculations of Potter and Morgan<sup>15</sup>. It also appears that  $\alpha$  is smaller in Ag compared to Cu: this cannot be explained with simple models which predict  $\alpha$  to be proportional to  $n^{-2/3}$  ( $n$  = electron density). This is discussed in more details in ref [12].

In order to look at the detailed behaviour of e-p scattering, we have subtracted  $v_{ee}$  from  $v(T)$  and plotted the rest ie  $v_{ep}$  divided by  $T^3$  on fig. 3. Drastic deviations from the usual  $T^3$ -law are observed. The low temperature part is well explained by ineffectiveness (equ. 7). The slow decrease of  $v_{ep}(T)/T^3$  with temperature above about 3K is due, we believe, to the energy dependence of the e-p SR. At low temperature, when the scattering is weakest, all electrons within a few times  $k_B T$  from the Fermi level will participate to the RFSE signal. However, when temperature increases e-p scattering will become stronger until eventually only the electrons with the lowest SR, that is the electrons at the Fermi level, will be able to contribute to the RFSE-resonance. A changing over will then occur from an energy averaged e-p SR to a smaller SR for electrons at  $\epsilon = \epsilon_F$ . As discussed in the review by Wagner and Bowers, this is included in the following expression for the amplitude of the RFSE-line, which takes into account the energy dependence of  $v_{ep}$ :

$$\langle A(T) \rangle = A(0) \cdot \int d\epsilon \cdot \left( -\frac{\partial f^0}{\partial \epsilon} \right) \cdot e^{-v_{ep}(\epsilon, T) \cdot t_c} \quad (8)$$

This expression contains no additional parameter. We have analyzed our Ag-data with a full expression for  $v_{ep}(T)$  using eqs. (8) and (7) and the result is shown in fig. 3. The agreement between the calculated curves and the experimental data is now excellent at high temperatures and is slightly less good than before for low frequency data at low temperatures.



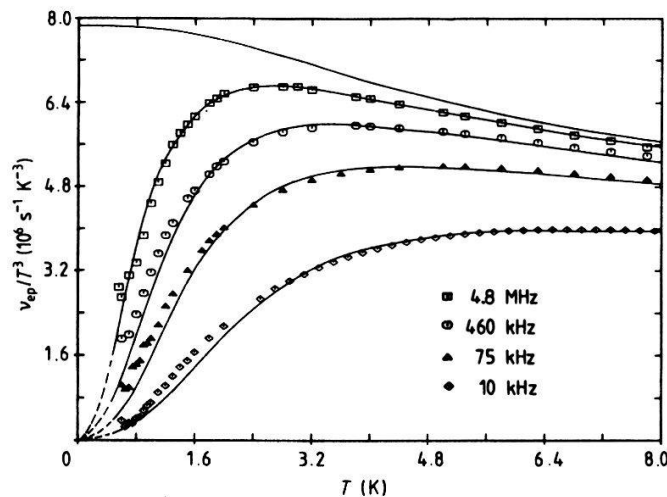


Figure 3:  $v_{ep}(T)/T^3$  for Ag with  $B//\langle 100 \rangle$  and  $d=1.32\text{mm}$ . The upper curve is calculated under the assumption of totally effective scattering down to 0K to show the effect of the energy dependence of the SR.

Below about 2K the experimental data are systematically somewhat above the calculated curves. Nevertheless these differences are quite small,  $< 2 \cdot 10^{-3}$  between the calculated and measured signal amplitudes in the case of the 10 kHz data, where the disagreement is strongest. This probably reflects a limitation of the model.

In Ag the large differences between published values (see Fig.1) are probably due to different experimental conditions. A reduction of frequency from 5 MHz to 10 kHz causes  $(\delta/d)^{1/2}$  to increase by 2.8, whereas a reduction of thickness from 1.3 mm (our Ag-sample) to 0.23 mm (Johnson's sample) increases  $(\delta/d)^{1/2}$  by 2.4. So their data should be close to our data taken at 10 kHz which is the case. So it appears that the 40% difference in the values of  $\beta$  reported in ref. [4] and [5] is consistent with our model.

In conclusion one can say that the RFSE has sufficient resolution to detect a small e-e SR in presence of a complicated e-p term whose linearity in  $T^3$  is usually only a gross simplification. Moreover it is possible by changing the RF by orders of magnitude to turn on or off the sensitivity to e-p collisions below about 1.5K which allows an unambiguous determination of the e-e term and gives access, at higher temperature, to the true e-p SR.

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