Compounded chaos

Autor(en): **Holzner, R. / Flepp, L. / Stoop, R.**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **62 (1989)**

Heft 6-7

PDF erstellt am: **22.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-116158

COMPOUNDED CHAOS

R.Holzner, L.Flepp, R.Stoop, G.Broggi, E.Brun
Physik -Institut der Universität Zürich, Switzerland

ABSTRACT

The NMR-Laser can be driven into chaos by either forced parameter modulation or delayed feedback. Distinct chaotic response is observed when both mechanisms act seperately or simultaneously. The corresponding strange attractors have been investigated experimentally and numerically with regard to fractal dimensions and Lyapunov exponents.

NMR-LASER AND CHAOS

The free NMR-Laser [1] is modelled by a three dimensional system of nonlinear differential equations (Bloch-Kirchhoff equations)

$$\frac{dB_u}{dt} = -k\cdot(B_u - B_k\cdot Q^{1/2}) - C\cdot M_v$$

$$\frac{dM_v}{dt} = -\frac{B_u}{T_2} + 9\cdot g\cdot M_z\cdot B_u$$

$$\frac{dM_z}{dt} = -\frac{(M_z - M_e)}{T_e} - 9\cdot g\cdot M_v\cdot B_u$$

with B_u the transverse magnetic field in the rotating frame, k the cavity decay constant, B_k the mean thermal noise field, Q the cavity quality factor, C the coupling constant, M_v the transverse nuclear magnetization, M_z the longitudional nuclear magnetization, M_e the equilibrium magnetization, T_2 the transverse relaxation time, T_e the longitudional relaxation time and g the gyromagnetic ratio.

To obtain chaotic behavior we either modulate the inhomogeniety of the static magnetic field which amounts at a modulation of T_2, effectively, or drive the quality factor Q of the resonator by a delayed feed back signal from the NMR-Laser output. The two mechanisms are described by

$$\frac{1}{T_2} = \frac{1}{T_{2c}} + A\cdot\cos(\Omega\cdot t)$$

$$Q = Q_c - B\cdot Q(t-T)\cdot|M_v(t-T)|$$

with Q_c the maximum cavity quality factor, B the feed back amplification constant, T_{2c} the effective transverse relaxation time, A the modulation amplitude, Ω the modulation frequency.

EXPERIMENT AND RESULTS

Applying both mechanisms simultaneously results in a chaotic response shown in fig.1a) which we call compounded chaos.

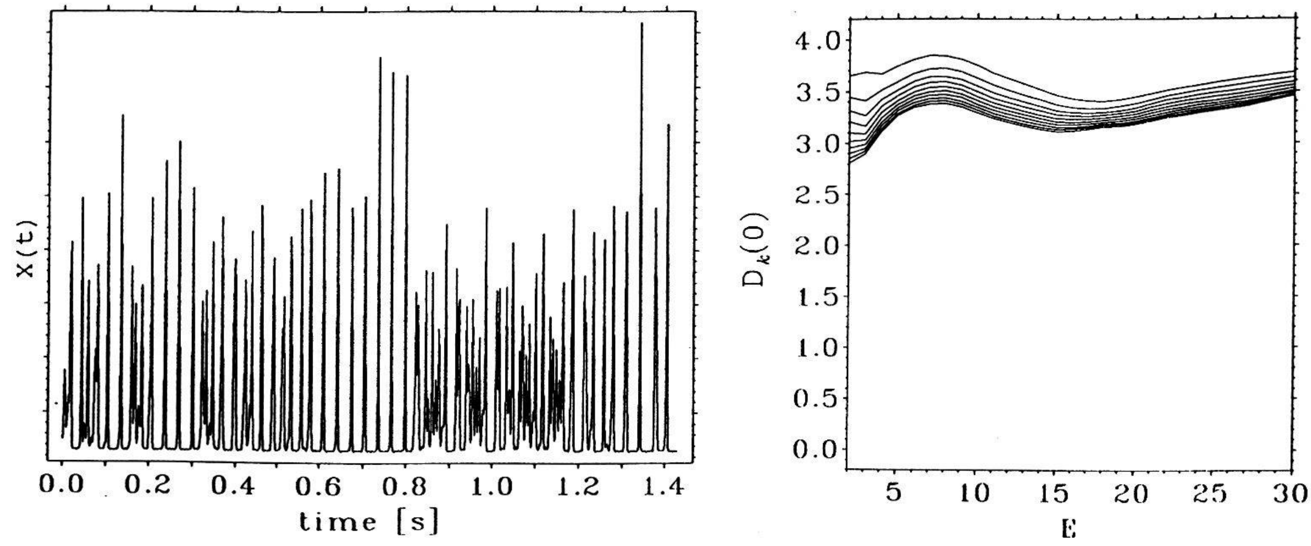

Fig.1 a) NMR-Laser output voltage x(t) proportional to the transverse nuclear magnetization M_y as a function of time for compounded chaos. Shown are 10^3 data points taken from a digitized time series of $5\cdot10^5$ data points sampled over a period of 700 s.

b) fractal information dimension D(0) as a function of embedding space dimension E and mass k=5,10,...,40 from top to bottom line.

We determined the fractal dimension numerically [2] from several sets of experimental time series of all three chaos mechanisms. For the set of which the compounded chaos is shown in fig.1, the value for the information dimensions is D(0)=2.57 +/- 0.07 for chaos by modulation, D(0)=3.15 +/- 0.1 for chaos by delayed feed back and D(0)=3.28 +/- 0.13 in the case of compounded chaos. The used numerical method produces excellent convergence of the fractal dimension and hence a small error . For compounded chaos the convergence is less obvious due to a missing broad plateau of constant D values as a function of E (fig.1b). Therefore a larger error had to be assigned to D.
The numerical determination [3] of Lyapunov exponents from experimental time series yields clearly one positive value in all cases which is expected for chaotic behavior. The occurence of a second positiv Lyapunov exponent has been found in the case of compounded chaos indicating hyperchaos.
The application of delayed feed back does not increase the dimension of the system by more than one. A larger increase would be possible in principle since the delay equation corresponds to an infinite number of differential equations with an infinite number of initial conditions. Chaos induced by delayed feed back seems to dominate the compounded state as far as the fractal dimension is concerned.

REFERENCES

[1] E.Brun, B.Derighetti, D.Meier, R.Holzner, M.Ravani,J.Opt.Soc.Am.**B2**,156 (1985)
[2] M.Ravani, B.Derighetti, G.Broggi, E.Brun, J.Opt.Soc.Am.**B**,1026 (1988)
[3] R.Stoop, P.F.Meier, J.Opt.Soc.Am.**B5** ,1037 (1988)