A mechanistic classical laboratory situation violating the Bell inequalities with 22, exactly "in the same way" as its violations by the EPR experiments

Autor(en): **Aerts, Diederik**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **64 (1991)**

Heft 1

PDF erstellt am: **22.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-116299

Helvetica Physica Acta
Vol. 64 (1991)

0018-0238/91/010001-23$1.50+0.20/0

A mechanistic classical laboratory situation violating the Bell inequalities with $2 \cdot \sqrt{2}$, exactly 'in the same way' as its violations by the EPR experiments.

Diederik Aerts*),
Theoretische Natuurkunde,
Vrije Universiteit Brussel,
Pleinlaan 2, 1050, Brussels.

(17. V. 1990, revised 31. VIII. 1990)

1. Abstract :

We present a macroscopical mechanistic classical laboratory situation, and a classical macroscopical entity, and coincidence measurements on this entity, that lead to a violation of the Bell inequalities corresponding to these coincidence measurements. The violation that we obtain with these coincidence measurements is exactly the same as the violation of the Bell inequalities by the well known coincidence measurements of the quantum entity of two spin 1/2 particles in a singlet spin state. With this we mean that it gives rise to the same numerical values for the expectation values and the same numerical value $2 \cdot \sqrt{2}$ for the expression used in the Bell inequality. We analyze the origin of the violation, and can formulate the main difference between the violation of Bell inequalities by means of classical entities and the violation of Bell inequalities by means of quantum entities. The making clear of this difference can help us to understand better what the quantum-violation could mean for the nature of reality. We think that some classical concepts will have to be changed, and new concepts will have to be introduced, to be able to understand the reality of the quantum world.

2. Introduction.

The violation of Bell inequalities by measurements on quantum mechanical entities is

*) Research Associate of the National Fund for Scientific Research (Belgium).

certainly one of the most stimulating happenings for the research on the physical meaning of the quantum formalism for the nature of reality. A lot of reflections have been made by many physicists and philosophers after the finding of Bell [1] that there was something 'really mysterious' about the structure of the correlations predicted by quantum mechanics for coincidence spin-direction measurements on a quantum entity consisting of two spin 1/2 particles in a singlet spin state. Meanwhile the structure of these correlations has been confirmed by measurements, which shows that the mystery is in the nature of the reality of these quantum entities.

Many profound and careful reasonings have been made about the possible significations for the nature of reality of the appearance of the structure of these correlations [2,3,4,5] but not any kind of agreement has been reached among the physicists working in the field [6].

A problem with 'theoretical reasonings' on such fundamental subjects, is that often unconsciously one uses hypothesis that are more or less hidden in the mathematics that goes along with the theoretical reasoning. These hypothesis seem perhaps obviously satisfied for the 'physical picture' that one has in mind of the situation in question, but are perhaps in general not so obviously satisfied in reality. With this principle idea in mind "Reality can be more complicated than one imagines at the first place", already some time ago we gave an example of a macroscopical physical entity that could violate the Bell inequalities [7,8]. We have improved the original example, and presented and analyzed it in different ways, with the aim of trying to understand more about the physical meaning of the violation of Bell inequalities by quantum entities for the nature of reality [9,10,11,12]. Our main aim was to present a macroscopical example violating Bell inequalities in the ordinary macroscopical reality, with the purpose to see "how near one can succeed in producing this strange structure of correlations in ordinary reality". Meanwhile we also have analyzed in detail what are the sometimes 'additionally unconsciously used assumptions' (we have called them AUUA) in the different derivations of the Bell (type) inequalities [13].

In the analysis of this example we have been considering the following form of the Bell inequality, originally derived by Bell in [14] :

$$| E(a,b) - E(a,b') | + | E(a',b) + E(a',b') | \leq 2 \tag{1}$$

where E(a,b) is the expectation value of the observable corresponding to a coincidence measurement m(a,b) on a entity S, where for outcomes {yes,yes} and {no,no} of the coincidence measurement we agree to give value +1 to the corresponding observable, and for outcomes {yes,no} and {no,yes} we agree to give value -1 to this observable. The same definition holds for the other quantities E(a,b'), E(a',b), and E(a',b').

These quantities where introduced by Bell, having in mind the quantum entity consisting of two spin 1/2 particles, produced in a singlet spin state. The spin part of the state vector of this entity in quantum mechanics is represented by the vector :

$$\Psi_S = \frac{1}{\sqrt{2}} [u_n \otimes u_{-n} - u_{-n} \otimes u_n] \tag{2}$$

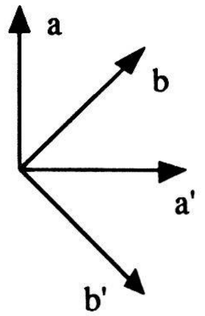
where **n** is a normalized vector in three dimensional Euclidean space, and u_n is the corresponding vector in the two dimensional complex Hilbert space that is used in quantum mechanics to describe the spin of a spin 1/2 particle. If $\mathbf{n} = (\cos\phi\cdot\sin\theta, \sin\phi\cdot\sin\theta, \cos\theta)$ then we can take $u_n = (e^{-i\phi/2}\cdot\cos\theta/2, e^{i\phi/2}\cdot\sin\theta/2)$. Remark that $-\mathbf{n} = (\cos(\pi+\phi)\cdot\sin(\pi-\theta), \sin(\pi+\phi)\cdot\sin(\pi-\theta), \cos(\pi-\theta))$ and hence $u_{-n} = (-i\cdot e^{-i\phi/2}\cdot\sin\theta/2, i\cdot e^{i\phi/2}\cdot\cos\theta/2)$. We have $\sigma\cdot n\ u_n = u_n$ and $\sigma\cdot n\ u_{-n} = -u_{-n}$, such that u_n and u_{-n} describe the states in which a particle has respectively spin 'up' and spin 'down' along the direction **n**. The singlet state is 'spherically' symmetric, **n** can be any direction in space, (2) always leads to the same vector Ψ_S.

The measurements considered by Bell are the ones originally proposed by Bohm [15)] in his reasoning on the EPR problem, and consist of making coincidence measurements of the spin on this entity consisting of two spin 1/2 particles in the singlet spin state along well defined directions of space.

If we calculate the expectation value E(a,b) of the observable corresponding to the coincidence measurement m(a,b) following the rules of quantum mechanics, we find

$$E(a,b) = \langle \Psi_S \mid \sigma\cdot a \otimes \sigma\cdot b \mid \Psi_S \rangle = -\,\mathbf{a}\cdot\mathbf{b} \tag{3}$$

It is very easy to see that for a good choice of the space directions **a**, **b**, **a'**, **b'**, the Bell inequalities are violated for these quantum mechanical expectation values. For example consider the situation as shown in (fig 1) then,



The choice of space directions to measure the spin, that allows a maximum violation of the Bell inequalities (fig 1).

we find :

$$\mid E(a,b) - E(a,b') \mid + \mid E(a',b) + E(a',b') \mid = 2\cdot\sqrt{2} \tag{4}$$

which violates the inequality. This is the maximum violation that quantum mechanics predicts for the inequalities. In the case of the macroscopical example that violates the inequalities, presented in [7,-,13] the violation is not $2\cdot\sqrt{2}$ as in the quantum case, but 4, which is more than the quantum violation. It seems at first sight a little bit strange that we can violate the Bell inequalities more with coincidence measurements on a classical macroscopical entity, than with coincidence measurements on the quantum entity. We shall understand why this is the case after the analysis that we will present in this paper. We shall see that in a certain sense the presence of the quantum probabilities temper the violation of the Bell inequalities.

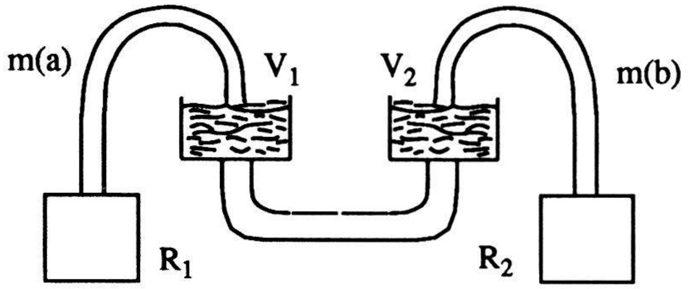
In this paper we want to present a macroscopical entity, violating the Bell inequalities, 'exactly in the same way' as the quantum entity of two spin 1/2 particles in the singlet spin state do. With 'exactly in the same way', we mean, as to the numerical value, but also as to the values of the expectation values of the different correlations. We could use the quantum formalism in its details to give a description of the macroscopical entity that we will present in this paper. This does of course not mean that we want to pretend that this is the way things happen with the spin entity. Not at all. Our aim is to limit of strictly the classical ways of violations of the inequalities, not by considering theoretical reasonings, but by looking at explicit examples of real physical entities in the ordinary reality.

In the second section we will present again the old example, because of its simplicity, together with the reasoning that tries to find out the AUUA, in the derivation of the Bell inequalities. In the third section we will present a classical spin-model, that we need to built our macroscopical entity that 'imitates' the violation of the inequalities by means of the quantum entity consisting of two particles in the singlet spin state. This classical spin-model has been presented earlier in [16,17]. In the fifth section we will present the example. In the sixth section we will analyze the example, and in the seventh section we will speculate about the possible meaning of it all. We want to repeat again that it is our main intention on the hand of these examples to see how far we can go, without giving up profound principles about the nature of reality, in creating situations that realize this strange form of correlations that lead to the violation of Bell inequalities.

3. The classical macroscopical example that violates Bell inequalities in classical reality.

The entity consists of two vessels V_1 and V_2 that contain each 10 litre of water and are connected by a tube. The measurement m(a) consists of taking the water out of V_1 with a siphon and collecting it in a reference vessel R_1. If we collect more than 10 litres, the outcome for m(a) is 'yes', and if we collect less than 10 litre the outcome for m(a) is 'no'. The measurement m(b) is the same as m(a) but performed on V_2. The coincidence measurement m(a,b) consists of performing m(a) and m(b) together. This coincidence measurement creates correlations. Indeed, if we find more than 10 litres in R_1, then we find less than 10 litres in R_2, and vice verse. The correlations are detected at both sides when the water stops flowing, this means simultaneously. Hence the events that correspond to the detection of the correlations are space-like separated events.To calculate Bell inequalities we have to introduce two other measurements.

The measurement m(a') consists of taking 1 litre of water out of V_1 and checking whether the water is transparent.



A classical macroscopical system that violates Bell-inequalities (fig 2).

If the water is transparent , then the outcome of m(a') is 'yes', and if it is not transparent the outcome is 'no'. The measurement m(b') is the same as m(a') but performed on V_2. We make coincidence measurements m(a,b'), m(a',b) and m(a',b'). We now define the following random variables : E(a) = +1 if m(a) gives 'yes', and E(a) = -1 if m(a) gives 'no'. In the same manner we define E(b), E(a'), and E(b'). We also define the random variables for the coincidence experiments, E(a,b) = +1 if m(a,b) gives "yes,yes" or "no,no", and E(a,b) = -1 if m(a,b) gives "yes,no" or "no,yes".

If the entity is in such a state that the two vessels of water contain 10 litre of transparent water, then E(a,b) = -1 , E(a',b) = +1 , E(a,b') = +1 , E(a',b') = +1. Hence :

$$| E(a,b) - E(a,b') | + | E(a',b) + E(a',b') | = +4 > +2. \tag{5}$$

This shows that Bell inequalities are violated, and if we compare with the quantum mechanical violation, we see that our entity violates the Bell inequalities 'more' than the quantum mechanical entity of the two spin 1/2 particles in the singlet spin state. If we present our second classical example in section 4, we will understand why this is the case.

In relation with this example we will now analyze the original derivation of the Bell inequalities using a locality hypothesis for the case of deterministic hidden variables. Later on the inequalities have been derived for the case of non-deterministic hidden variables, but since in any way in this macroscopical example the hidden variables are deterministic, we need not consider these generalizations at this moment. We will see that also this locality hypothesis is violated for the case of our example.

A deterministic hidden variable theory is a theory that postulates the existence of states of the entity such that all observables have a determined outcome when the state is known. Let us denote by Γ the set of these states λ. Hence in such a theory, E(a,b) has a determined outcome E(a,b, λ) for every state λ. Bell introduces then the following hypothesis :

Bell locality hypothesis : *For all measurements m(a), m(b), and m(a,b) and for all λ we have:*

$$E(a,b,\lambda) = E(a,\lambda)\cdot E(b,\lambda) \tag{6}$$

The physical meaning of this hypothesis is that the result of the measurement m(a) only depends on the state λ and not on the measurement m(b). Since this Bell locality hypothesis implies Bell inequalities to be satisfied, our example must also violate the Bell locality hypothesis. Let us try to see why this is so. It is very easy to specify the deterministic hidden variables for our entity. Indeed, if we specify for example the diameters λ_1 and λ_2 of the two siphons, the outcomes of all the measurements are determined. Hence we can write :

$$E(a,b,\lambda_1,\lambda_2) = E(a,\lambda_1,\lambda_2)\cdot E(b,\lambda_1,\lambda_2), \tag{7}$$

$$E(a,\lambda_1,\lambda_2) = +1 \text{ and } E(b,\lambda_1,\lambda_2) = -1 \text{ if } \lambda_1 > \lambda_2 \tag{8}$$

$$E(a,\lambda_1,\lambda_2) = -1 \text{ and } E(b,\lambda_1,\lambda_2) = +1 \text{ if } \lambda_1 < \lambda_2 \tag{9}$$

This is a correct factorisation if one performs the coincidence measurement m(a,b). If one wants however to use the same E(a, λ_1, λ_2) to factorize the random variable E(a,b') from the coincidence measurement m(a,b') it does not work any more. Indeed, m(a) performed together with m(b') always gives the outcome 'yes'. This means that the value of E(a) does not only depends on the states λ_1, λ_2 but also on the fact that we perform measurement m(b) or m(b'), and

$$E(a,\lambda_1,\lambda_2,b) \neq E(a,\lambda_1,\lambda_2,b') \tag{10}$$

since E(a, λ_1, λ_2, b) = +1 if $\lambda_1 > \lambda_2$ and E(a, λ_1, λ_2, b) = -1 if $\lambda_1 < \lambda_2$ while E(a, λ_1, λ_2, b') = +1 for all λ_1, λ_2.

Bell has put forward this locality hypothesis having in mind the entity consisting of two spin-1/2 particles in the singlet spin state. Why do people find this locality hypothesis 'natural' for this entity? Because they imagine the entity to be an entity consisting of two spin-1/2 particles located in different widely separated regions of space while they are flying apart. And indeed, for two entities located in widely separated regions of space, with no connection between them, the

Bell locality hypothesis seems to be a natural hypothesis to be satisfied. But for two entities that actually form a whole (like the water in the two vessels) it is very easy to violate the Bell locality hypothesis, and hence also the Bell inequalities.

We would now like to find out the physical reason for the violation of the Bell inequalities by our macroscopical entity. We can already understand very much if we consider the nature of the hidden variables λ_1 and λ_2. These are not hidden variables of the state of the entity 'water', before the measurement, because the state of the water (and hence the reality of the water) is completely determined by the fact that the volume is 20 litre. And λ_1 (λ_2) is a hidden variable of the measurement m(a) (m(b)), but not of the measurement m(a') (m(b')). Hence if we would analyze this situation in the scheme of 'non classical probability models', as we have done in [16,17], we would classify the hidden variables λ_1 and λ_2 as representing 'hidden measurements', and not 'hidden states', and as is shown in [18], the water as in the example has a 'non classical' probability model. This explains from a probabilistic point of view why we can violate Bell inequalities with our example of the vessels of water. Indeed, the correlations that are detected by the measurement m(a,b) were not present before, but are **created** during the measurement, and therefore they can violate the Bell locality hypothesis. We propose to call correlations that were not present before the measurement and are created by and during the measurement **'correlations of the second kind'**. Correlations that were already present before the measurement and are only detected by the measurement, we will call **'correlations of the first kind'**.

Let us give an example of such correlations of the first kind. Consider a entity consisting of two material point particles moving in space and having total momentum zero. A coincidence measurement of the momenta of the individual particles gives us correlated results. These correlations were however already present before the coincidence measurement. The measurement only detects the correlations and does not create them. These kinds of correlations can never be used to violate Bell inequalities, because the result of a measurement on one of the particles will never depend on what measurement is being performed on the other particle. If we read the paper on the history of the EPR paper of Max Jammer in [19], it becomes clear that this difference was exactly the point that puzzled Einstein, and was at the origin of the EPR article.

Let us try to summarize : If we consider correlations that are created by and during the coincidence measurement m(a,b) (correlations of the second kind), then it is possible to violate Bell inequalities and the Bell locality hypothesis by means of this coincidence measurement and some other measurement, because the outcome of measurement m(a) will in general depend on whether we perform m(a) together with m(b) or with some other measurement m(b'). If we consider correlations that were already present before the coincidence measurement, then the Bell locality hypothesis will be satisfied, and Bell inequalities cannot be violated.

Statement : *The possibility of violating Bell inequalities is not only a property of quantum-entities. Bell inequalities can also be violated by coincidence measurements on a classical macroscopical entity. In fact Bell inequalities can always be violated if during the coincidence experiments one breaks one entity into separated pieces, and by this act creates the correlations. In analogy with the example of the vessels of water, a lot of other macroscopical entities violating Bell inequalities can be invented. But there is of course no mystery in these violations, because we see with our own eyes, inside our own human reality what happens.*

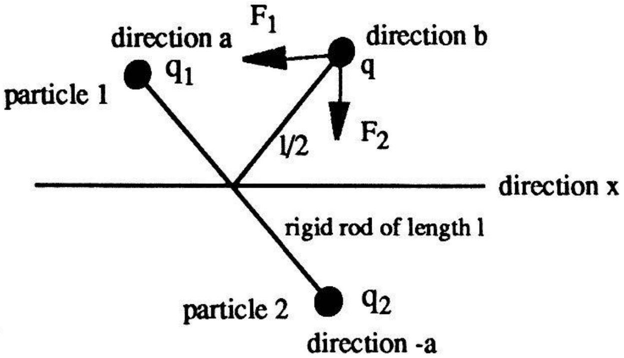
Let us now introduce a macroscopical model for the spin of a spin 1/2 particle, and then construct the macroscopical example that 'imitates' the quantum way of violating the Bell inequalities.

4. A macroscopical classical model for the spin of a quantum particle.

The classical macroscopical spin model that we will present in this section has been presented in [16,17] with the aim of giving a possible explanation for the the non-classical character of the quantum probability model. It is shown in [16,17] that a lack of knowledge about the measurements on a physical entity gives rise to a non classical probability calculus for this physical entity. It is also shown that the non classical probability calculus of quantum mechanics can be interpreted as being the result of a lack of knowledge about the measurements. It is as a specific example of such a classical model giving rise to a quantum mechanical probability calculus, that the spin model that we will present now, is introduced in [16,17]. In [16] we have constructed an example by using charges that can move under the influence of the Coulomb force on the surface of a sphere. In [17] we have considered masses that move under the influence of the gravitational force. We want to point out here, that the aspect of using charges or masses, makes the example physically real, but is not the most important part of it. The important point is that we can present a 'mechanistic' example, constituted of particles that move under influence of interactions in our ordinary three dimensional Euclidean space. Hence in the presentation of the example here, we will put full attention to this aspect. Let us give the model.

The classical macroscopical system that we consider is a particle characterized by a parameter q (in [16] this parameter q was taken to be a fixed positive charge, in [17] it was taken to be a fixed mass, but this is of no essential importance for the mechanistic aspect of the example). We will from now on indicate this particle by the parameter q, and speak of the particle q.

We give a detailed description of the measuring apparatus and the measurements. We have a rigid rod of a certain length l (see fig 3). At the end-points of the rod are two particles. One particle is characterized by a parameter q_1 , and we will call it particle q_1, and the other particle is characterized by a parameter $Q - q_1 = q_2$ where Q is a fixed parameter and we will call it particle q_2 (negative charges in [16]) and masses in [17])). The rigid rod is placed fixed in the laboratory such that particle q_1 is in space direction **a**, and particle q_2 in space direction **-a** in a plane orthogonal to some fixed direction **x**. The particle q can be put in the neighbourhood of the measurement apparatus, and will then be attracted by the particle q_1 and q_2 of the measuring apparatus.

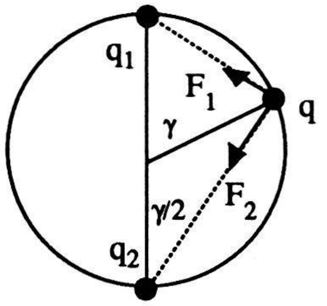


The measuring apparatus consists of a rigid rod of length l with at the endpoints two particles, particle q_1 and particle q_2. The measurement m(a) consists of letting the particle q move under the influence of the forces of attraction and be captured by one of the two particles of the measurement apparatus. If it is captured by particle q_1 then we give outcome "a-up" to measurement m(a), if it is captured by particle q_2 we give outcome "a-down" to the measurement m(a) (fig 3).

We suppose that this happens in a viscous medium, such that under the influence of friction, finally the particle q will end up at the place of one of the particles q_1 or q_2 of the measuring apparatus. If it ends up at particle q_1 we give the outcome "**a**-up", and if it ends up with particle q_2 we give the outcome "**a**-down" for the measurement m(**a**).

We can now start making repeated measurements with measurement m(**a**) on particle q in a state which is determined by a direction **b** and length l/2 from the centre of the measuring apparatus (see fig 3). We can count the number N(**a**-up) of outcomes "**a**-up" or the number N(**a**-down) of outcomes "**a**-down" and divide by the total number N of particles q that have participated in the repeated measurements. If the relative frequencies ν(**a**-up) and ν(**a**-down) approximate real numbers between 0 and 1 if N goes to infinity, then we call these real numbers the probabilities P(**a**-up) and P(**a**-down). We can introduce the following probabilities : P(**a**, **b**) = the probability that if particle q is in state of direction **b**, and the measurement m(**a**) is performed, the outcome "**a**-up" will occur. P(-**a**, **b**) = the probability that if particle q is in state of direction **b**, and the measurement m(**a**) is performed, the outcome "**a**-down" will occur.

To determine these probabilities, we should go to a laboratory and perform such repeated experiments, and then see what we find for the relative frequencies. We can also work out the 'mechanistic model' and then use our knowledge of classical mechanics to calculate the probabilities. Let us regard the measurement situation of our classical macroscopical example a little bit closer, and see which model we can propose. The three particles are located in a plane, particle q in a point indicated by the direction **b**, and particles q_1 and q_2 in diametrically opposed points indicated by the directions **a** and **-a** (see fig 4). Let us call γ the angle between



We consider the three particles from fig 3 as they are located in a plane, particle q is located in the point indicated by the direction b, particle q_1 and q_2 of the measuring apparatus are located in the points indicated by the directions a and -a. F_1 and F_2 are the forces of attraction between q_1 and q and q_2 and q (fig 4).

the two space directions **a** and **b**. We propose the following mechanistic model :

1) The forces F_1 and F_2 of attraction are proportional to the product of the parameters characterizing the particles divided by the square of the distance between the particles (these would be Coulomb forces if q, q_1, and q_2 represent charges, like in ref 16, and gravitational forces if q, q_1, and q_2 represent masses as in ref 17). Hence introducing a constant C we have :

$$|F_1| = C\cdot \frac{q_1\cdot q}{l^2\sin^2(\gamma/2)} \qquad |F_2| = C\cdot \frac{q_2\cdot q}{l^2\cos^2(\gamma/2)} \tag{11}$$

2) The particle q moves under influence of the two forces F_1 and F_2 of the measurement apparatus, and finally will arrive at rest at one of the two places indicated by direction **a** or **-a**, depending on the magnitude of the forces of attraction between the three charges. If $|F_1|$ is bigger than $|F_2|$ particle q will move, and arrive at the place of particle q_1. If $|F_1|$ is smaller than $|F_2|$ particle q will move, and arrive at the place of particle q_2 .

3) The parameters q_1 and q_2 are arbitrary, their only constraint being that their sum must equal the fixed parameter Q. This situation can be modelled by supposing that q_1 is an at random number in the interval [0,Q], and $q_2 = Q - q_1$. By means of these hypothesis 1, 2 and 3, we can make a mathematical derivation for the probabilities P(**a**, **b**) :

$$P(\mathbf{a}, \mathbf{b}) = \text{Probability that } |F_1| \text{ is bigger than } |F_2|$$

$$= P\left(C\cdot \frac{q_1\cdot q}{l^2\sin^2(\gamma/2)} > C\cdot\frac{q_2\cdot q}{l^2\cos^2(\gamma/2)}\right)$$

$$= P\,(q_1\cos^2(\gamma/2) > q_2\sin^2(\gamma/2))$$

$$= P\,(q_1\cos^2(\gamma/2) > (Q - q_2)\,\sin^2(\gamma/2)) \;= P\,(q_1 > Q\,\sin^2(\gamma/2))$$

$$= \frac{Q - Q\cdot\sin^2(\gamma/2)}{Q} = \cos^2(\gamma/2). \tag{12}$$

This is exactly the probability that we would find if m(**a**) represented the measurement of the spin of a spin 1/2 particle in the **a** direction while the particle has spin in the **b** direction.

We can describe this macroscopical system by the formalism of quantum mechanics. Every state of the particle q in the direction **b** represented by spherical coordinates (θ, ϕ) is represented by the unit vector

$$u_b = (e^{-i\phi/2}\cdot\cos(\theta/2), e^{i\phi/2}\cdot\sin(\theta/2)) \text{ with } \mathbf{b} = (\cos\phi\cdot\sin\theta, \sin\phi\cdot\sin\theta, \cos\theta) \quad (13)$$

of a two dimensional complex vector space as is well known for the spin of a spin 1/2 particle. And the measurement m(**a**) where $\mathbf{a} = (\cos\beta\cdot\sin\alpha, \sin\beta\cdot\sin\alpha, \cos\alpha)$ is represented by means of the self-adjoint operator

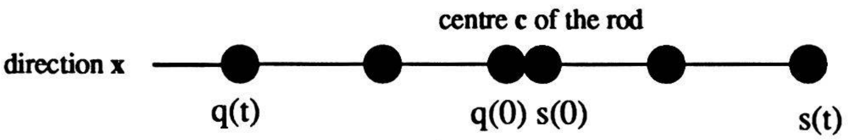
$$S_{\alpha\beta} = \begin{pmatrix} \cos\alpha & e^{-i\beta}\cdot\sin\alpha \\ e^{i\beta}\cdot\sin\alpha & -\cos\alpha \end{pmatrix} \quad (14)$$

The eigenvalue +1 corresponds to the outcome "**a**-up" of the measurement m(**a**) and the eigenvalue -1 to the outcome "**a**-down".We remark again that the state of the particle q is a pure state and the probability only comes from a lack of knowledge about the measurement, or with other words, the hidden variables are in the measurement, and not in the state of the system. These hidden variables are the values of the parameters q_1 and q_2.We want to remark that other hypothesis about the details of the mechanistic model can lead to other types of non-classical probability models, that in certain occasions can be shown to be also non-quantum probability models (see ref 16 for details on this aspect). In this paper, we only want to use this spin model for the construction of a macroscopical classical entity that imitates the violation of the Bell inequalities of the quantum entity consisting of two particles of spin 1/2 in the singlet spin state. So we are not really interested how, and by which forces the classical motion is governed, as long as the model that we propose remains purely mechanistic.

5. The classical macroscopical example that imitates the quantum mechanical violation of the Bell inequalities.

We shall now construct our classical macroscopical entity that violates Bell inequalities in exactly the same way as does the quantum mechanical entity of the two spin 1/2 particles in the singlet spin state.

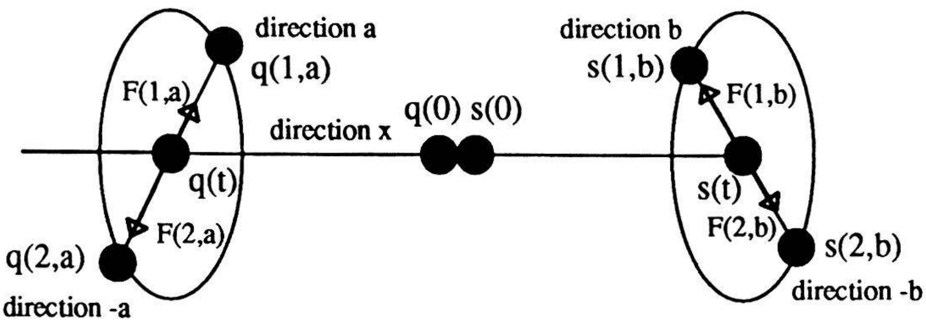
The entity consists of two particles characterized by parameters q and s (as we remarked in the foregoing section we could take two positive charges, or two masses, for these two parameters) that are bound on a rigid rod along the direction **x**, and move on this rigid rod in the following way. At time t = 0 the two particles are in the centre **c** of the rod, and they move outwards, q to the left and s to the right (see fig 5).



The system of two particles q and s bounded on a rigid rod in direction **x**, that move outwards, q to the left and s to the right (fig 5).

The coincidence measurements are constructed on the hand of analogue measurements than the measurements m(**a**) introduced in the foregoing section. This means that at the two ends of the rigid rod, we have measuring apparatuses constructed as described in the foregoing section. And the rigid rods used for the construction of the measuring apparatuses are in a plane orthogonal to the direction **x** (see fig 6).

Let us call A the circle with radius l/2, the collection of points where the particles q(1, a) and q(2, a) of the left measuring apparatus can be located, and B the circle with radius l/2, the collection of points where the particles s(1, b) and s(2, b) of the right measuring apparatus can be located. For the measurement apparatus to perform measurement m(**a**) at the left we choose at random in an interval [0,Q] the parameters q(1,a) and q(2,a) such that q(1,a) + q(2,a) = Q, and locate them on the rigid rod of the measurement apparatus, this means on the circle A, in the points **a** and **-a**.

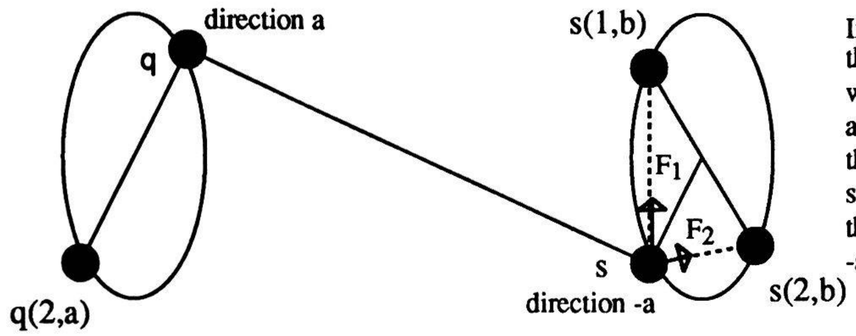


The system of two particles q and s that move outwards on a rigid rod in direction x, together with the measuring apparatuses, that consists of diametrically located particles q(1,a) and q(2,a) on a circle A at the left in a plane orthogonal to x, and diametrically located particles s(1,b) and s(2,b) on a circle B at the right, in a plane orthogional to x (fig 6).

For the measurement apparatus to perform the measurement m(**b**) at the right, we choose at random in an interval [0,S] the parameters s(1,b) and s(2,b) such that s(1,b) + s(2,b) = S, and locate them on the rigid rod of the measurement apparatus, this means on the circle B, in the points **b** and **-b**. The two measurement apparatuses are at equal distances of the centre **c** of the rigid rod on direction **x**, the place where the two particles q and s were at time t = 0. At time t the two particles q and s have arrived at the centres of the two circles A and B.

If the two particles have arrived at the centres of the two circles, we can consider the following forces. For the measurement apparatus at the left we consider two forces F(1,a), the force between q and q(1,a), and F(2,a) the force between q and q(2,a). For the measurement apparatus at the right we consider two forces F(1,b), the force between s and s(1,b), and F(2,b) the force between s and s(2,b) (see fig 6). In this sense once the particles q and s have arrived at the centres of the circles A and B, there are four forces F(1,a), F(2,a), F(1,b) and F(2,b) that are considered. We now perform the measurement m(**a**, **b**) which is the performance of m(**a**) and m(**b**) together.We propose the the following mechanistic model for this measurement :
One of the four forces is the biggest one in magnitude. Suppose that this biggest force is at the left, hence it is F(1,a) or F(2,a). If F(1,a) is bigger than F(2,a), the particle q will move from the centre of the circle A towards the particle q(1,a), and finally remain at rest in the point **a** of the circle A. If F(1,a) is smaller than F(2,a), the particle q will move from the centre of the circle A towards the particle q(2,a), and finally remain at rest in the point **-a** of the circle A. We suppose that the rigid rod that connects the two particles q and s, can rotate freely around its centre **c**. Hence by the motion of particle q, the motion of the particle s will be determined as well. If q arrives at the point **a** of circle A, s will arrive at the point **-a** of the circle B. If q arrives at the point **-a** of the circle A, s will arrive at the point **a** of the circle B. Because the particles q(1,a) and q(2,a) are chosen at random in the interval [0,Q] we obviously have probability 1/2 that F(1,a) is bigger than F(2,a) and hence the particles q and s move as shown in fig 7, and probability 1/2 that F(1,a) is smaller than F(2,a) and hence the particles q and s move as shown in fig 8.

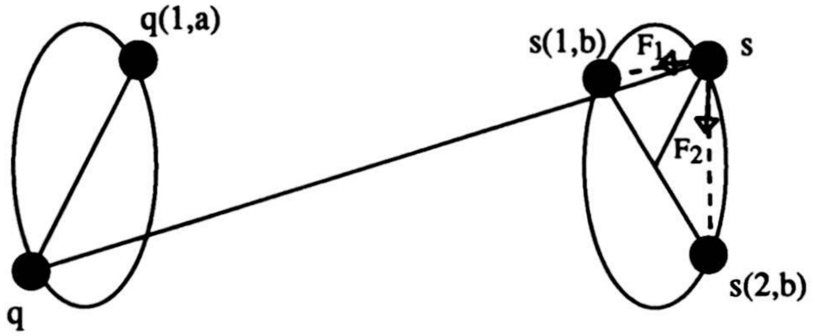
Till this moment the particles q and s have been connected by being attached to the rigid rod. We make now the hypothesis that once the particles q and s touch the circles A and B, the connection breaks down. The particle q will not move any further, since it is already at point **a** or at



If the force F(1,a) is bigger than the force F(2,a) the particle q will move towards the particle q(1,a) and arrive finally at the point a of the circle A. The particle s that is still connected with q by means of the rigid rod, will arrive at the point -a of the circle B (fig 7).

point **-a** of circle A.The particle s will however move further on, since it is still worked on by the force F_1 between s and s(1,b) and the force F_2 between s and s(2,b) (see fig 7,8).

We are in a situation that has been treated in detail in the foregoing section and make the same hypothesis than the one we have made in foregoing section for the nature of the mechanistic evolution that follows. If F_1 is bigger than F_2 the particle s will move towards s(1,b), and finally



If the force F(1,a) is smaller than the force F(2,a) the particle q will move towards the particle q(2,a) and arrive finally at the point -a of the circle A. The particle s that is still connected with q by means of the rigid rod, will arrive at the point a of the circle B (fig 8).

arrive in the point **b** of circle B. If F_1 is smaller than F_2 the particle s will move towards s(2,b), and finally arrive in the point **-b** of circle B. The corresponding probabilities with which these events happen are $\cos^2(\gamma/2)$ and $\sin^2(\gamma/2)$ where γ is the angle between the point where s is located on the circle B and the point where s(1,b) is located as can be calculated (see 12).

If the biggest of the four forces is at the right, the symmetric mechanical motions as the ones just explained are supposed to happen.

We have now described a measurement m(**a**, **b**) of which the possible outcomes are the following (see fig 6) :

1) Outcome (a,b) which means that q arrives at **a**, and s arrives at **b**.
2) outcome (a,-b) which means that q arrives at **a**, and s arrives at **-b**.
3) outcome (-a,b) which means that q arrives at **-a**, and s arrives at **b**.
4) and outcome (-a,-b) which means that q arrives at **-a** and s arrives at **-b**.

We will now calculate the probabilities for the measurement m(**a**, **b**) to give the different possible outcomes.

Let us calculate the probability P(a,b) for outcome (a,b) to occur. In the case that the biggest of the four forces is at the left, we have probability 1/2 that q arrives at point **a** of circle A, and hence s arrives at point **-a** of circle B and then probability $\cos^2(\gamma/2)$ that s arrives at point **b** of circle B where γ is the angle between **-a** and **b**. Hence the probability that the measurement m(**a**, **b**) gives outcome (a,b) which means particle q at point **a** of circle A and particle s at point **b** of circle B, is given by $1/2\cdot\cos^2(\gamma/2)$ where γ is the angle between **-a** and **b**. If we denote the angle between **a** and **b** by arc(a,b), and remark that $\text{arc}(a,b) + \gamma = \pi$ then we find :

$$P(a,b) = 1/2\cdot\sin^2(\text{arc}(a,b)/2) \qquad (15)$$

And in a similar way we find :

$$P(a,-b) = 1/2\cdot\cos^2(\mathrm{arc}(a,b)/2)$$
$$P(-a,b) = 1/2\cdot\cos^2(\mathrm{arc}(a,b)/2)$$
$$P(-a,-b) = 1/2\cdot\sin^2(\mathrm{arc}(a,b)/2) \qquad (16)$$

In the case that the biggest of the four forces is at the left, from reasons of symmetry immediately follows that we will find the same probabilities P(a,b), P(a,-b), P(-a,b) and P(-a,-b). If we construct now the random variable corresponding to the measurement m(**a**, **b**), that consists of giving value +1, if we have an outcome (a,b) or (-a,-b), and value -1, if we have outcome (a,-b) or (-a,b), then the expectation value E(a,b) of this random value can easily be calculated.

$$\begin{aligned} E(a,b) &= (+1)\cdot\{P(a,b) + P(-a,-b)\} + (-1)\cdot\{P(a,-b) + P(-a,b)\} \\ &= \sin^2\left(\frac{\mathrm{arc}(a,b)}{2}\right) - \cos^2\left(\frac{\mathrm{arc}(a,b)}{2}\right) \\ &= -\cos(\mathrm{arc}(a,b)) \\ &= -\,a\cdot b \end{aligned} \qquad (17)$$

This is exactly the same expectation value than the one calculated for the random variable introduced by Bell in relation with the coincidence spin measurements on the entity consisting of two spin 1/2 particles in the singlet spin state, as we can see in (3).

If we now make the same choices for **a**, **b**, **a'**, and **b'** as proposed in fig 1, then the expectation values E(a,b), E(a, b'), E(a',b), and E(a', b') corresponding to the measurements m(**a**, **b**), m(**a**, **b'**), m(**a'**, **b**), and m(**a'**, **b'**) will violate the Bell inequalities as calculated in (4).

A remark must be made :

As we have described our classical macroscopical measurement m(**a**, **b**), one could say that it is not exactly a coincidence measurement, since the particle q at the left (or the particle s at the right, depending on which of the four forces is the biggest one) reaches the point a or -a (or the point b or -b), and then the particle s at the right still has to start moving towards the point b or the point -b (or the particle q at the left still has to start moving towards the point a or -a). This is true. The following answers can be given. For reasons of symmetry, it is obvious that a real measurement of the type that we have proposed, will lead to coincidence outcomes. But the simple mechanical model that we have proposed looses in a certain sense this symmetry. We could consider a more complicated mechanistic model, that does not loose this symmetry, by considering the motion of the particles q and s, taking into account the four forces F(1,a), F(2,a), F(1,b, and F(2,b) at once. If we do this we find however a very complicated problem of classical mechanics, that in principle is solvable, but not in an easy way. Bell inequalities will still be violated. We have however introduced our simplification also for another reason. As we shall see, it is this simplification that is very close to the 'algebraic calculations' on the quantum mechanical situation.

6. Discussion of the macroscopical entity and further analogy with the quantum case.

6.1. If we compare the two examples, the classical macroscopical example with the vessels of water, and the classical macroscopical example of the particles moving outwards on the rigid rod, we see that the reason why in the case of the vessels of water we find a stronger violation of the Bell inequalities, is due to the fact that this example is completely deterministic. It are the hidden variables present in the measuring apparatus that 'soften' in a certain sense the violation of the Bell inequalities in the example of section 5. Hence paradoxically, one can violate Bell inequalities more severely by means of purely deterministic classical entities that form a whole and are then broken apart, than by means of entities that form a whole and then are broken apart and are not deterministic. The presence of quantum-like probabilities 'softens' the violation.

6.2 The detailed analysis that we have presented in section 3 remains completely valid for the example of section 5. Indeed also here the violation is produced by the presence of what we have called **correlations of the second kind**. It is the possibility of the rigid rod that connects particles q and s, to rotate around its centre **c**, that correspond to the possibility of the quantum particle to have a spin. The direction in which this rigid rod rotates correspond to the direction of spin. And indeed, at the moment that the two particles q and s reach the centres of the circles A and B, the rigid rod that connects them has not yet rotated around its centre in any direction. Only at this moment the property that correspond in our model to the spin, is starting to get created. Since the two particles q and s are still forming one entity, because connected by the rod, the 'joint creation' of this property 'spin' will give rise to correlations of the second kind that typically violate Bell inequalities.

6.3 Let us regard a little bit more in detail the calculation in quantum mechanics of the probabilities P(a,b) :

$$P(a,b) = \langle \Psi_S \mid P_a \otimes P_b \mid \Psi_S \rangle \qquad (20)$$

where P_a and P_b are the projections on the vectors u_a and u_b of the two dimensional complex Hilbert space that represents the spin states. If we take into account (2) and the fact that Ψ_S does not depend on the direction n, we have :

$$
\begin{aligned}
P_a \otimes P_b(\Psi_S) &= \frac{1}{\sqrt{2}}(P_a \otimes P_b)\cdot(u_a \otimes u_{-a} - u_{-a} \otimes u_a) \\
&= \frac{1}{\sqrt{2}}(u_a \otimes P_b(u_{-a})) \\
&= \frac{1}{\sqrt{2}}\cdot \sin\left(\frac{\mathrm{arc}(a,b)}{2}\right)(u_a \otimes u_b) \qquad (19)
\end{aligned}
$$

and since in general for any projector P in a Hilbert space, and any unit state vector Ψ of this Hilbert space we have

$$
<\Psi \mid P \mid \Psi> = < P(\Psi) \mid P(\Psi) > \qquad (20)
$$

we can write

$$
\begin{aligned}
<\Psi_S \mid P_a \otimes P_b \mid \Psi_S> &= < P_a \otimes P_b(\Psi_S) \mid P_a \otimes P_b(\Psi_S) > \\
&= \frac{1}{2}\cdot \sin^2\left(\frac{\mathrm{arc}(a,b)}{2}\right)\cdot <u_a \otimes u_b \mid u_a \otimes u_b> \\
&= \frac{1}{2}\cdot \sin^2\left(\frac{\mathrm{arc}(a,b)}{2}\right) \qquad (21)
\end{aligned}
$$

We present this calculation because in the algebraic steps of the calculation we can see a similarity with the 'real' happenings in our macroscopical example. First the superposition singlet state is reduced to a product state by the projection operators, and then the squares of the corresponding projection distances deliver the probabilities.

Let us now try to see that the 'description' of quantum mechanics 'indicates' that we are in a situation of correlations of the second kind. This we can see by analyzing the form of the singlet spin state of the entity consisting of two particles. We already remarked that this state as presented in (2) is independent of the direction of the vector n. But there is more. In fact only one such state can be made. Indeed also for different directions a, b in space we have that :

$$
\Psi_S = k\,(u_a \otimes u_b - u_b \otimes u_a) \text{ for some complex number } k \qquad (22)
$$

This shows that Ψ_S does not depend on any direction of any vector in space. It is just a mathematical construction, using these vectors, that lead to a unique vector in the tensor product Hilbert space. Hence Ψ_S does not represent a state of two particles which have already their spin, although mathematically it is constructed in this way. It represents a state of an entity, consisting of two particles, which do not have yet their spins. And the spins are created by the coincidence measurement, that is exactly the measurement that takes apart the one entity, and breaks it into to two 'separated' entities. In our example the situation is exactly the same. The state of the two particles q and s when they are in the centres of the two circles A and B is such a 'singlet' state. The rigid rod has no rotation around its centre s till this moment. The rotation is created from that moment on.

7. Is the concept of 'entities moving around and interacting inside space' at stake ?

The two classical examples that violate Bell inequalities, that we have presented here, are of course not 'physical models' for the real happening of the violation of the inequalities with the spin entity. But they are in a certain sense 'philosophical models'. We can see by means of these examples that there is something to explain in relation with the violation of Bell inequalities. The only possible conclusion, compatible with a philosophical attitude of realism, seems to be to accept that the entity of two quantum particles of spin 1/2 in the singlet spin state is an entity of non-separated entities, till the moment that the coincidence measurement starts to 'separate' it.

This would mean that this one entity is present in a part of space of a macroscopical magnitude (of the order of 12 meters of length in the case of Aspects experiments). Is it imaginable that two quantum-entities can form one whole in such a huge region of space? For the water, there is no problem, because we can put a connecting tube, as long as we want, and for the charges we can put a connecting rigid rod as long as we want, but for two quantum-entities?

Moreover, for example in the Aspect measurement, the space like parts of the photons, if described by wave-packets, seem to fly apart, because they pass through two filters of different frequencies. Hence, although it is very complicated to make a rigourous description of these space like parts, they seem to be 'separated' in a certain sense. While all the 'non-local' aspects are due to the spin-like parts of the entities. From this follows that we should not only 'imagine' ourselves a photon-pair with the dimensions of a 'cloud' of 12 meter diameter, but with the additional fact, that there is 'nothing' (we have to say 'nothing' in ordinary space) between. The particles remain 'one whole' while the space regions with probabilities of detecting one of the particles almost equal to 1 get separated at macroscopical distances. This is certainly a situation that we will not be able to imitate by means of a classical macroscopical entity.

Indeed, if we consider two space regions R_1 and R_2 that are macroscopicaly separated and an entity S that constitutes a whole such that in the region R_1 we have a probability almost equal to 1 to detect one part S_1 (or S_2) of the entity S, and in the region R_2 we have a probability almost equal to 1 to detect the other part S_2 (or S_1) of the entity S. And such that between the two regions we have a probability almost equal to 0, to detect the entity, then this situation can only be realized by means of a macroscopical entity when this macroscopical entity is already separated into two sub entities. Macroscopical entities seem to have one additional property, that microscopical entities not necessarily have. Let me call it the 'property of macroscopical wholeness'.

The property of macroscopical wholeness : *For macroscopical entities we have the following property : if they form a whole (hence are not two separated parts), then they hang together through space. Which means they cannot be localized in different macroscopicaly separated regions R_1 and R_2 of space, without also being present in the region of space 'between' these separated regions R_1 and R_2 .*

To understand clearly this property of macroscopical wholeness, think of the example of the water. If we cut of the connecting tube, the two vessels of water get localized in different separated regions of space, but then they are separated entities, and Bell inequalities cannot be violated any more with coincidence measurements on them. Microscopical entities seem to be able to constitute a whole, without necessarily being submitted to this property of macroscopical wholeness. This is in my opinion what we have to learn from the EPR experiments as a matter of fact.

If it were only the EPR measurements that cause us troubles of understanding about the nature of reality, we could still hope to find a solution in the sense of questioning principles like 'causality, or Einstein locality etc..'. But other measurements, ever more strongly, and more clearly, indicate that quantum-entities do not necessarily satisfy this property of macroscopical wholeness. In fact it seems to be a very common thing for quantum entities. Together with J. Reignier we are investigating the situation of a typical Stern-Gerlach measurement in relation with this problematic [20,21]. And we are trying to understand the reality of the state of such 'one' particle, when it comes out of the Stern-Gerlach magnet. Our aim is to work out the meaning of this state showing that this one particle (hence only one entity) effectively does not satisfy this property of macroscopical wholeness. In a certain sense it is 'detectable' in two separated regions R_1 and R_2 and not between.

It is very difficult for us to imagine that one entity as a whole does not satisfy this property of macroscopical wholeness. But this difficulty probably comes exactly from the part of the reality of our macroscopical space-concept, that we humans have constructed, from our experiences with macroscopical entities that all do have this property. As we humans imagine space, and the entities that are in it, we think that only two situations are possible.

Situation 1 : *An entity forms a whole, and then breaking it into parts can make us perform measurements that violate Bell inequalities, but then this entity must 'hang' together, and hence cannot be localized in different separated regions of space.*

Situation 2 : *An entity is formed out of two separated entities, and then making experiments on the parts never will make it possible to violate Bell inequalities. In this case the separated parts can of course be localized in different separated regions of space.*

Other situations are very difficult for us to imagine. We repeat that in my opinion this comes from the fact we have constructed space from our human experiences with macroscopical entities, that exactly satisfy this macroscopical wholeness property.

We can speculate and invent a scenario of how the classical world-image of space filled up with macroscopical entities that interact with each other 'inside' this space could have grown, the macroscopical entities satisfying the macroscopical wholeness principle, although originally the entities of which the macroscopical entities in it are constructed do not satisfy this macroscopical wholeness principle. A speculation of this kind is presented at the end of [13]. There will be needed however a lot of investigation to be able to see in detail which aspects of the human construction of the classical world-image with its macroscopical entities are to be relaxed to be able to 'understand' the reality of the quantum world. An analysis of this kind is started in [22]. Such an analysis does not however have to start from nothing. We want to remark that already for more than thirty years formalisms have been created with the main purpose of 'explaining' the quantum theory, but all being founded on concepts more fundamental than the classical concepts of entities made up of 'substance' and interacting 'inside' a three dimensional Euclidean space. I myself have been participating in the elaboration of one of these formalisms, originated by J.M. Jauch, founded by C. Piron, and now commonly called the Geneva-formalism [23,24,25]. In this type of formalisms it must be possible to encounter and introduce the concepts necessary for the development of such a physics, really detached from some of the old classical images, making impossible an understanding of the quantum world till now. But a lot of work remains to be done.

To end this section we want to make one additional remark on this classical wholeness principle, that is also in a subtle way connected to the problematic of non-locality touched upon in this paper. The two aspects, the one of non-locality treated in this paper, and the one of 'incompleteness' of the quantum theory, both were presented in the original EPR paper. There is the following fact : If one wants to describe a collection of quantum entities by the mathematical formalism of ordinary quantum mechanics, then it can be shown, that such entities can never be separated in the classical sense (hence situation 2 cannot be described). This is a shortcomings of the mathematical structure (the vector space structure of its set of states, hence the superposition principle) of the ordinary quantum formalism. By considering a more general formalism, as for example the Geneva formalism , this shortcoming can be investigated in detail, as has been done in [7,24,26]. As is shown in [7,24,26] and more specifically in [27] it is this shortcoming that is at the origin of the 'logical' content of the EPR paper. An entity consisting of two separated quantum entities cannot be described by the quantum formalism, since its collection of states cannot be represented in a vector space. Hence what Einstein Podolsky and Rosen claim in their paper, namely that the quantum formalism is 'incomplete' in the sense that it cannot represent all elements of reality of an entity consisting of two separated quantum entities is correct (as explained by Max Jammer in [19] this is probably the part of the content of the EPR paper due to Boris Podolsky, and his contacts with Kurt Gödel). In [27] we point out explicitly the missing elements of reality. They are however not missing states, as implicitly suggested by EPR and explicitly claimed by some physicists who understood the EPR reasoning as a reasoning indicating a kind of incompleteness (similar to the incompleteness of thermodynamics) that can be solved by adding hidden variables to the description of the states, and in this way introducing more states. The incompleteness of quantum theory is of a much more subtle nature, as is explained in [27].

8. Conclusion.

If one studies in detail the example of the mechanistic macroscopical laboratory situation, violating the Bell inequalities 'in the same way as' the EPR experiments violations, than one can notice that a lot of similar situations can be created. The necessary requirements to create such a situation are :

To consider two entities that form a whole till a certain moment where they are separated by the actions of the coincidence measurements. This separation produces two separated entities in a 'product' state of the type $p_a \cdot p_{-a}$, $p_{-a} \cdot p_a$,or $p_{-b} \cdot p_b$, $p_b \cdot p_{-b}$ such that at the right the realization of one of the product states $p_a \cdot p_{-a}$, or $p_{-a} \cdot p_a$ happens with probability 1/2, or at left the realization of one of the product states $p_{-b} \cdot p_b$ or $p_b \cdot p_{-b}$ happens with probability 1/2. The transition to the final product state $p_a \cdot p_b$ must then happen with a probability equal to the $\cos^2$ of the half angle between a,b. This is exactly a situation that we have created in section 5. But obviously it is possible to invent other equivalent situations. It would be nice to construct along these lines a machine that really could be built in the laboratory, to create this quantum-like violation of the Bell inequalities.

9. References.

1) Bell, J., Physics 1,195 (1964).

2) Clauser, J.F. and Shimony, A., Reports on Progress in Physics 41, 1881 (1978).

3) d'Espagnat, B.*"A la Recherche du Réel"*, Gauthiers-Villars, Paris (1979).

4) Bell, J., *"Bertlmann's socks and the nature of reality"*, in *"Les implications Conceptuelles de la Physique Quantique"*, Journal De Physique, 42 (1981).

5) See the articles in *"Symposium on the Foundations of Modern Physics"*, eds.Lathi P. and Mittelstaedt P., World Scientific, Singapore (1985), and in *"Microphysical Reality and Quantum Formalism"*, eds. van der Merwe A. et al, Kluwer Academic Publishers (1988).

6) Dutch, W. and Aerts, D. *"The great quantum poll"*, Physics Today, 39, 13 (1986).

7) Aerts, D., "The One and the Many", Doctoral thesis, TENA, Vrije Universiteit Brussel (1981).

8) Aerts, D., Lett. Nuovo Cim. 34, 107 (1982).

9) Aerts, D., *" How do we have to change quantum mechanics in order to describe separated systems? "* in *"The Wave-Particle Dualism"*, eds. Diner S. et al., Reidel, Dordrecht (1984).

10) Aerts, D., *" The physical origin of the Einstein Podolsky Rosen paradox "* in *"Open Questions in Quantum Physics"*, eds. Tarozzi G. and van der Merwe A., Reidel, Dordrecht (1985).

11) Aerts, D., *"The physical origin of the EPR paradox and how to violate Bell inequalities by macroscopic systems"* in *"Symposium on the Foundations of Modern Physics"*, eds.Lathi P. and Mittelstaedt P., World Scientific, Singapore (1985).

12) Aerts, D., *"The description of separated systems and quantum mechanics and a possible explanation for the probabilities of quantum mechanics"* in *"Microphysical Reality and Quantum Formalism"*, eds. van der Merwe A. et al, Kluwer Academic Publishers (1988).

13) Aerts, D., *"An attempt to imagine parts of the reality of the micro-world"*, to appear in the proceedings of the conference *"Problems in Quantum Physics ; Gdansk '89'"* to be published by World Scientific Publishing Company, Singapore (1990).

14) Bell, J., *"Foundations of Quantum Mechanics"*, ed. B. d'Espagnat, New-York; Academic (1971).

15) Bohm, D., "Quantum Theory", Prentice-Hall (1951).

16) Aerts, D., J. Math. Phys. 27 , 202 (1986).

17) Aerts, D., *"The origin of the non-classical character of the quantum probability model"*, in *" Information, complexity, and control in quantum physics"*, edited by A. Blaquiere, S. Diner, and G. Lochak, Springer-Verlag, Wien-New York, (1987).

18) Aerts, D., *"A possible explanation for the probabilities of quantum mechanics and a macroscopic situation that violates Bell inequalities"* in *" Recent Developments in Quantum Logic "*, eds. Mittelstaedt P. and Stachow E.W. in "Grundlagen der Exakter Naturwissenschaften", band 6, Wissenschaftverlag, Bibliografisches Institut, Mannheim (1985).

19) Jammer, M., in *"Symposium on the Foundations of Modern Physics"*, eds.Lathi P. and Mittelstaedt P., World Scientific, Singapore (1985).

20) Aerts, D. and Reignier, J., *" A quantum mechanical treatment of the Stern-Gerlach experiment."*, in preparation.

21) Aerts, D. and Reignier, J.,*" The spin of a micro-entity and problems of non-locality"*, submitted for the conference on the Foundations of Modern Physics, Joensuu, Finland, August (1990).

22) Aerts, D. "The construction of reality", in preparation.

23) Piron, C. " *Foundations of Quantum Physics* ", W. A. Benjamin, Inc. London, Amsterdam (1976).

24) Aerts, D. " *Description of many physical entities without the paradoxes encountered in quantum mechanics* ", Found. Phys. 12, 1131 (1982).

25) A recent overview of the formalism with the necessary references can be found in, Piron, C. " Recent Developments in Quantum Mechanics ", Helv. Phys. Acta, 62, 82, (1989).

26) Aerts, D., *"The description of one and many physical systems"* in *"Les Fondements de la Mécanique Quantique"* 63, edited by Christian Gruber, A.V.C.P. Lausanne (1983).

27) Aerts, D., Helv. Phys. Acta 57, 421 (1984).