Non-existence of path space measure for local (QED)_1

Helvetica Physica Acta
Vol. 64 (1991)

0018-0238/91/050548-09$1.50+0.20/0

Non-existence of path space measure for local $(\mathrm{QED})_1$*

By J. Löffelholz

Karl Marx University, Department of Physics/NTZ, 7010 Leipzig, Germany

(16.X.1990)

Abstract. We study the interaction of a "charged" particle with an oscillator. On the classical level holds $m\dot{x} = p - eA$, where $x, A \in \mathbb{R}$. In QM we let x move on the circle S to have a proper ground state. The imaginary time Green's functions exist, satisfy OS-like axioms and, for $e \neq 0$, are complex valued. They define a normalized quasimeasure $d\lambda$ on path space $Q \times A$. Our main result is the proof of $\|\lambda\| = +\infty$, due to a theorem of Yngvason. Integrating out the oscillator variable A we find some probability measure $d\mu$ on Q (given by the effective action for the particle). Because of memory it allows us to recover the Hamiltonian semigroup for the coupled quantum system.

1. Introduction

On a heuristic level the idea of path integral was introduced by Feynman [1]. After reformulation of QFT in terms of Euclidean Green's functions [2] its existence became a challenge for mathematicians [3].

In particular Yngvason [4] obtained the following result: Given those Green's functions then strong OS-positivity implies that a measure exists and must be real. He used an argument of Fröhlich. However from QED we know that the interaction of charged matter with gauge fields is given by a complex phase factor and, if Θ denotes time reflection, one has combined $PC\Theta$-symmetry. To understand the crux we looked for some caricature of electromagnetism in standard QM avoiding any troubles with Fermions [5].

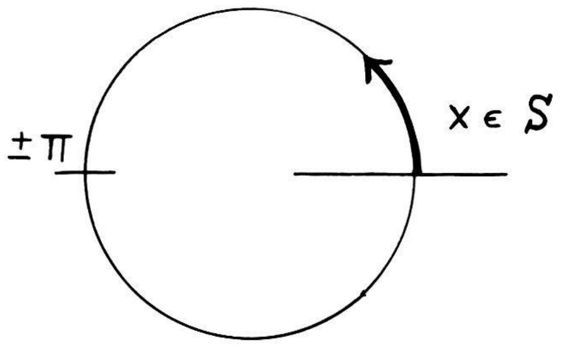

Fig. 1.

*) Seminar given at the Workshop "white noise analysis: New results and their impact on quantum physics", ZiF Bielefeld Sept. 24–29, 1990

Let x be the angular coordinate of a particle moving on the circle as drawn above. The stationary states of the system are described by eigenfunctions $\Psi = \exp\{i \cdot (kx)\}$, $k \in \mathbb{Z}$, of $p = -id/dx$. Indeed imposing periodic boundary conditions at $x = \pm\pi$ the momentum p defines a selfadjoint operator [6] in $L^2(S)$. When x couples to a homogeneous magnetic field of flux $\Phi = 2\pi A$ then p acquires a shift by $\alpha = eA$, where e was the charge. We claim that for $\alpha \notin \mathbb{Z}$ (otherwise the effect would be unvisible) the propagator leads to a complex-valued normalized cylinder measure $d\omega_\alpha$ of infinite total variation [7].

So contrary to general believe the formal substitution $t \to -it$ does not resolve all problems with the path integral.

2. Model

Below we consider $A \in \mathbb{R}$ as dynamical degree of freedom describing a quantum oscillator. On the classical level our model is given by the coupled equations of motion

$$\left.\begin{aligned} m\ddot{x} &= eE \\ \ddot{A} + \beta^2 A &= e\dot{x} \end{aligned}\right\}, \tag{1}$$

where $E = -\dot{A}$. Clearly, $p = m\dot{x} + eA$ and total energy H are conserved. We will fix m equal to one. In QM we realize

$$H = \frac{(p - eA)^2}{2} + I_\beta(E, A), \tag{2}$$

where $I_\beta = 1/2(E^2 + \beta^2 A^2)$, as a Hermitean operator in the separable Hilbert space $\mathscr{H} = L^2(S \times \mathbb{R})$. Of course $p \in \mathbb{Z}$ so that H has discrete spectrum and a proper ground state Ω. The variable $x \in S$ gives a bounded operator with norm $\|x\| = \pi$. This affects the classical identity $m\dot{x} = p - eA$. Indeed, we find the singular anomalous commutation relations

$$\begin{aligned} L &= px - xp \\ &= i \cdot \sum_{k \neq 0} (-1)^k e^{ikx}, \end{aligned} \tag{3}$$

and hence

$$[H, x] = \frac{1}{2i}\{(p - eA)L + L(p - eA)\}. \tag{4}$$

3. Propagator

To obtain the propagator one may start from the Lagrangean [8], calculate the action along a trajectory $t \to (x(t), A(t))$, $t_1 \leqslant t \leqslant t_2$, and then go to imaginary time. Because of the restriction $x \in S$ this seems to be a doubtful venture. Instead we rewrite

$$P^t = \exp(-tH)$$
$$= \exp\left(\frac{-tp^2}{2M}\right) \cdot V^* K^t V, \quad t \geqslant 0, \tag{5}$$

where $V = \exp\{ie/\gamma^2(pE)\}$, $\gamma = \sqrt{\beta^2 + e^2}$ and M is an effective mass. Moreover we introduced $K_\gamma^t = \exp(-tI\gamma)$, governing the oscillator [9]. The unitary V commutes with momentum p. So if

$$L^2(S \times \mathbb{R}) = \bigoplus_{k \in \mathbb{Z}} \mathscr{H}_k, \tag{6}$$

on wave functions $\Psi = \Psi(x, A)$ from some fixed sector $\mathscr{H}_k$ the operator V induces a shift of A to $A_k = A - ek/\gamma^2$. Using Poisson's formula [10] we get

$$P^t = \sum_{l \in \mathbb{Z}} \frac{\exp\left(-\frac{z_l^2}{2\tau}\right)}{\sqrt{2\pi\tau}} \cdot K_\gamma^t(A, B), \tag{7}$$

where

$$z_l = (y - x) + i\delta(t)\,\frac{e(A + B)}{2} + 2\pi l, \tag{8}$$

$\tau(t) = \gamma^{-2} \cdot (\beta^2 t + e^2 \delta)$ and $0 \leqslant \delta(t) \leqslant 2/\gamma$. Hence, for $e \neq 0$, in the Schrödinger representation the imaginary time propagator of the model $(\mathrm{QED})_1$ is complex-valued. We may hardly associate a genuine stochastic process with trajectories $t \to (x(t), A(t))$ on path space

$$Q \times A = \bigtimes_{t \in (-\infty, \infty)} (S \times \mathbb{R}). \tag{9}$$

4. Quasimeasure

Let us renormalize the Hamiltonian so that $H\Omega = 0$ and perform a unitary transformation on $L^2(S \times \mathbb{R})$ which brings the ground state vector Ω into the function equal one.

Then for any $t_0 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_n$ the iteration of the propagator defines a normalized complex-valued measure $d\lambda_n(xA, yB)$ on the space $\bigtimes_{j=0,1,\ldots,n} (S \times \mathbb{R})$. For $n = 1$ we obtain

$$d\lambda_1 = d\rho(xA) \cdot P^\varepsilon(xA, yB)\, dy\, dB, \tag{10}$$

where $\varepsilon = t_1 - t_0$ and $d\rho = \Omega^2 \cdot dx\, dA$ is the measure on $S \times \mathbb{R}$ defining the new scalar product in the "physical" Hilbert space. Its extension to some σ-additive measure on $Q \times A$ yields a nontrivial problem. Unfortunately we cannot use [11]. But the simple structure of our model allows us to control the total variation of $d\lambda_n$ in the limit $n \nearrow \infty$, directly.

We return to the situation of a particle x moving on the circle S in presence of magnetic flux Φ. The evolution operator

$$R_\alpha^t = \sum_{k \in \mathbb{Z}} \frac{e^{ik(y-x)}}{2\pi} \cdot \frac{e^{-t(k-d)^2}}{2} \frac{1}{2\pi} \sum_{k \in \mathbb{Z}} \exp\left\{ik(y-x) - \frac{t}{2}(k-\alpha)^2\right\}, \quad t \geqslant 0, \quad (11)$$

satisfies $R_\alpha^t 1 = \exp\left(-\frac{t\alpha^2}{2}\right) \cdot 1$, where $\alpha = e\Phi/2\pi$ and the Chapman equation. Via the Kolmogorov construction we find a normalized cylinder measure $d\omega_\alpha$ on Q. We claim that for given $\varepsilon > 0$

$$Y_\alpha(\varepsilon) = \exp\left(\frac{\varepsilon\alpha^2}{2}\right) \cdot \int_S |R_\alpha^\varepsilon(x, y)| \, dy \geqslant 1. \quad (12)$$

Equality holds if and only if $\alpha \in \mathbb{Z}$. By inspection the QM amplitude $(\cos x, R_\alpha^\varepsilon \sin x)$ is not real. We remark that $R_\alpha^\varepsilon(x, y)$ coincides with Jacobi's theta function [12] at $z = (y - x) + i\varepsilon\alpha$ and imaginary time parameter. Using the fact that the value of the integral does not depend on $x \in S$ we get

Lemma.

$$\|\omega_\alpha\|(S \times S^n) = Y_\alpha(\varepsilon)^n, \quad n = 0, 1, 2, \ldots \quad (13)$$

For large n the total variation of $d\omega_\alpha$ diverges. Conversely, let us fix $t > 0$, so that $\varepsilon = t/n \searrow +0$ if n tends to infinity. Then from $Y_\alpha(\varepsilon) \leqslant \exp(\varepsilon\alpha^2/2)$ and translation invariance we conclude that on a finite time interval the total variation stays bounded. We easily check Daletzki's condition. We observe the symmetry $d\omega_\alpha \cdot \Theta = (d\omega_\alpha)^*$. Complex conjugation is equivalent to changing α by $-\alpha$. How our lemma can be applied to the full propagator?

For small $\varepsilon \geqslant 0$ one may substitute $P^\varepsilon(xA, yB)$ by $R_\alpha^\varepsilon(x, y) \cdot K_\gamma^\varepsilon(A, B)$, with $\alpha = e \cdot (A + B)/2$. In the case $n = 1$ for the total variation of $d\lambda$ on $\times_{j=0,1} (S \times \mathbb{R})$ we obtain approximately

$$\iint_{\mathbb{R}^2} Y_{e(A+B)/2}(\varepsilon) \cdot d\varphi_\gamma(A, B), \quad (14)$$

where $d\varphi_\gamma$ denotes the oscillator measure. Finally we combine the above estimate with $e \cdot (A + B)/2 \notin \mathbb{Z}$, for a.e. $(A, B) \in \mathbb{R}^2$, and the fact that $d\varphi_\gamma$ was normalized. Similarly we proceed when $n = 2, 3, \ldots$. Hence $d\lambda$ acquires unbounded total variation. We shortly write $\|\lambda\| = +\infty$.

5. OS-axioms

If $e = 0$ the Hamiltonian is the sum of $P^2/2m$ and I_β. Its ground state is given by the vector $1 \otimes \Omega_\beta \in L^2(S \times \mathbb{R})$, where Ω_β stands for the oscillator vacuum.

At imaginary time we have a two-dimensional Markov process $t \to (x(t), A(t))$ on the large probability space

$$\left(Q \times A, \sum \times \mathfrak{A}, d\omega_0 \otimes d\varphi_\beta\right). \quad (15)$$

The measure $d\omega_\alpha$, for $\alpha = 0$, describes free Brownian motion in the circle S and $d\varphi_\beta$ governs an Ornstein-Uhlenbeck process. The moments factorize. Using $x \in S$ and the fact that $d\varphi_\beta$ is Gaussian we easily derive the estimate

$$\left|\langle x(s_1)x(s_2)\cdots x(s_m)\cdots A(t_n)\rangle_0\right| \leqslant \pi^m \frac{(n!)^{1/2}}{(2\beta)^n}, \tag{16}$$

valid for $m, n = 0, 1, 2, \ldots$. Of course for odd m or n this vanishes. As an exercise let us calculate the correlation function of $d\omega_0$. Expanding $f(x) = x$ in a Fourier series on $(-\pi, \pi)$ we get

$$\langle x(s_1)x(s_2)\rangle_0 = \sum_{k \neq 0} k^{-2} \cdot \exp\left(-\varepsilon k^2/2\right), \tag{17}$$

where $\varepsilon = |s_2 - s_1|$. In the limit $\varepsilon \searrow +0$ we recover the variance of the normalized Lebesque measure $dx/2\pi$ on S.

We denote $x = x(0)$ and $u = dx(t)/dt$. Then by the Feynman–Kac formula $-\langle x \cdot u(\varepsilon)\rangle_0$ for small ε becomes equal to the divergent expression

$$\tfrac{1}{2}(x\Omega, p^2(x\Omega)) = +\infty. \tag{18}$$

Because of $x \in S$ also in the interacting case the existence of Green's functions is rather trivial. But, as we learned above, their time derivatives are singular at coinciding arguments [13].

Theorem I. *The moments* $\langle x(s_1)x(s_2)\cdots x(s_m)\cdots A(t_n)\rangle$, *where* $m, n = 0, 1, 2, \ldots$ *of the cylinder measure* $d\lambda$ *on* $Q \times A$ *exist and are*

(i) *integrable,*
(ii) *time translation invariant,*
(iii) OS-*positive,*
(iv) *complex for* $e \neq 0$. (19)

Proof. Within the famous reconstruction theorem of Osterwalder and Schrader (iii) is a consequence of QM □. We would like to check it looking just at the Green's functions of our model. The idea is simple.

The propagator which defines $d\omega_\alpha$ satisfies $R_\alpha^s(x, y)^* = R_\alpha^s(y, x)$, where $x, y \in S$ and $s \geqslant 0$. So for any bounded function $f = f(x(s))$ we get

$$\int_Q f^* \cdot \Theta f \, d\omega_\alpha = \exp\left(-s\alpha^2\right) \cdot \|\Psi\|^2 \geqslant 0, \tag{20}$$

where $\Psi = R_\alpha^s f \in L^2(S)$. We also check the inequality for cylinder functions say $f = f(x(s_1), \ldots, x(s_n))$ with $s_1, s_2, \ldots, s_n$ in $\mathbb{R}_+$. Strong OS-positivity requires it to hold for any exponential function measurable with respect to

$$\Sigma_+ = \sigma\Bigg(\bigcup_{s \geqslant 0} \Sigma_s\Bigg). \tag{21}$$

We observe that $\sigma(\Sigma_- \cup \Sigma_+) = \Sigma$, where Σ_- is the image of Σ_+ under reflection Θ.

This generalizes to the cylinder measure $d\lambda$. What about Yngvason's result? He proved that the above conditions are not compatible with the existence of $d\lambda$ as a *finite* measure. So the upper bound

$$\frac{1}{z}\iint_{Q\times A} |x(s_1)x(s_2)\cdots x(s_m)\cdots A(t_n)|\, d\omega_0 \otimes d\varphi_\beta \tag{22}$$

on the Green's functions, for all $m, n = 0, 1, 2, \ldots$, implies $Z = 0$. Indeed the free measure $d\omega_0 \otimes d\varphi_\beta$ is ergodic and hence [14] $d\lambda$ cannot be the perturbation by some phase factor.

6. White noise

We remark that

$$\left\langle \exp\left\{i\cdot\left(\sum_{j=1}^{m} k_j x(s_j)\right)\right\} A(t_1)\cdots A(t_n)\right\rangle, \tag{23}$$

for $k_1, k_2, \ldots, k_m \in \mathbb{Z}$, has an integral representation with respect to the measure $dx \otimes d\eta(u, A)$, where $t \to u(t) \in \mathbb{R}$ for $e = 0$ was white noise [15]. More precisely, let us consider the operator

$$C = G_\gamma^{1/2}\begin{bmatrix} \Delta & -ie \\ -ie & 1 \end{bmatrix} G_\gamma^{1/2} \tag{24}$$

in $D = L^2(\mathbb{R}) \oplus L^2(\mathbb{R})$. Above $\Delta = \beta^2 - d^2/dt^2$ and $G_\beta(\cdot, \cdot)$ will denote the kernel of its inverse Δ^{-1}. One easily shows that C defines the covariance of the desired cylinder measure on $U \times A$. Of course the restriction of $d\eta$ to $\mathfrak{A}$ should coincide with $d\varphi_\gamma$. To avoid confusion we introduce another symbol $\langle\!\langle\cdot\rangle\!\rangle$ for expectation with respect to $d\eta$. We find

$$\langle\!\langle u(b)A(a)\rangle\!\rangle = -ie(b, G_\gamma a), \tag{25}$$

$$\langle\!\langle (u(b)A(a))^2\rangle\!\rangle = -2e^2\cdot(b, G_\gamma a)^2 + \left(b, \frac{1}{1+e^2G_\beta}\, b\right)(a, G_\gamma a), \tag{26}$$

etc. Provided $|e|^2 \leqslant \beta^2/2$, the last expression becomes non-negative. We claim that C is a sectorial operator [16] in D with half opening angle arctg (e/β). There is a striking analogy to the path space measure for the bosonized massless Schwinger model $(\mathrm{QED})_2$ [17]. Formally $d\eta$ is given by

$$e^{-z^2/2}\, dz \otimes d\varphi_\gamma\,|_{z = u + ieA}\,. \tag{27}$$

Of course the variable z was nothing but the imaginary time counter part of the canonical momentum $p = m\dot{x} + eA$, where m is fixed to one.

Let $t \to \xi(t) \in \mathbb{R}$ be one-dimensional Brownian motion mastering at time zero the slalom $\xi(0) \in (-\pi, \pi)$. This defines a Markov process on (X, Ξ, dv), where X is the space of trajectories and Ξ the σ-algebra generated by cylinder sets.

$$dv = dx \otimes e^{-u^2/2}\, du, \quad x = \xi(0), \tag{28}$$

is an averaged conditional Wiener measure. If we close $(-\pi, \pi)$ to the circle and consider the real line as covering space [18] of S we may identify periodic sets

$$\pi^{-1}(M) = \bigcup_{l \in \mathbb{Z}} \{\xi \in X : \xi(t) - 2\pi l \in M\}, \tag{29}$$

for Borel M in S and $t \neq 0$, with elements of Σ. In other words π was the canonical projection from $\mathbb{R}$ onto $S = \mathbb{R}/\mathbb{Z}$. The lift π^{-1} in a natural way induces a measure isomorphism. All that generalizes to the coupled system.

Theorem II.

$$d\lambda = dx \otimes d\eta \,|_{\pi^{-1}(\Sigma \times \mathfrak{A})}. \tag{30}$$

We emphasize that $d\lambda$ is translation invariant whereas the measure $dx \otimes d\eta$ was not [19]. The above redefinition allows us to calculate expectation values as $\langle \exp\{i(kx)\} \cdot A(t)\rangle$, for $k \in \mathbb{Z}$. Integrating over $x \in S$ we obtain $\delta(k)$ and then we are left with a Gaussian. For non-integer k everything becomes more tricky. But "Gott kümmert sich nicht um unsre mathematischen Schwierigkeiten. Er integriert empirisch" [20].

7. Memory

As hidden in the title we have an alternative resolution to the problem of existence of a path space measure for $(\mathrm{QED})_1$. We claim that in the mixed representation where x and E are diagonal the QM propagator $P^t = \exp(-tH)$, $t \geqslant 0$, is positivity preserving [21].

Indeed given any bounded $f(x, E) \geqslant 0$, because of $\Omega(x, E) \geqslant 0$, the vector $\Psi = f \cdot \Omega$ is also represented by a non-negative function in the physical Hilbert space $L^2(S \times \mathbb{R})$. We now apply the factors V, K_γ^t, V^* and $\exp\{-t(p^2/2M)\}$ step by step. V shifts the variable x to $x + eE/\gamma^2$, V^* conversely. With the other two operators there is no trouble. Hence $P^t\Psi(x, E) \geqslant 0$. In particular this will be true for any $\Psi = (g \otimes 1) \cdot \Omega$ with $g(x) \geqslant 0$, $x \in S$. By induction

$$(\Omega, g \otimes 1\, e^{-(t_2 - t_1)H} g_2 \otimes 1 \cdots g_n \otimes 1 \cdot \Omega) \geqslant 0, \tag{31}$$

provided $g_j(x) \geqslant 0$ for all $j = 1, 2, \ldots, n$. Let $\mathscr{M}$ denote the Abelian algebra of those bounded multiplication operators $F = g \otimes 1$, $\|F\| < \infty$, acting in $L^2(S \times \mathbb{R})$. It is the completion of

$$\mathscr{M}_0 = \{F = e^{ikx} \otimes 1 : k \in \mathbb{Z}\} \tag{32}$$

in norm and not maximal [22]. Instead we observe that the vacuum $\Omega = 1 \otimes \Omega\gamma$ is cyclic for the algebra $\mathscr{B}_0$ generated by P^t, $t \geqslant 0$, and the elements of $\mathscr{M}_0$. Of course $\mathscr{B}_0$ is dense in the algebra $\mathscr{B} = \mathscr{B}(L^2(S \times \mathbb{R}))$ of all bounded operators.

We claim that

$$\Psi(t) = P^t(e^{ikx} \otimes 1)\Omega, \tag{33}$$

$t \geqslant 0$, span $\mathscr{H}_k$ except in the case when $k = 0$. But the vacuum sector N of $L^2(S \times \mathbb{R})$ is spanned by the vectors $\Psi(t) = F^* P^t F \Omega$, $t \geqslant 0$, with any $F \in \mathscr{M}_0$ different from the unit element. This has a nice consequence.

Theorem III. *The triple*

$$L^2(S \times \mathbb{R}), \mathscr{M}_0, \{P^t, t \geqslant 0\} \tag{34}$$

together with the vacuum Ω *builds a generalized positive semigroup structure. Hence the above* QM *amplitude define the Fourier transform of a probability measure* $d\mu$ *on* Q.

Proof. See Klein's theorem [23] □. One can show that $d\mu$ is OS-positive. Given $0 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_n$ and $k_j \in \mathbb{Z}$ we find

$$\int_Q \exp\left\{i\left(\sum_{j=0}^n k_j x(t_j)\right)\right\} d\mu = \delta(k) \cdot \exp\left\{-\frac{1}{2}\left(\sum_{i,j=1}^n \sum k_i k_j \tau^{ij}\right)\right\}, \tag{35}$$

where

$$\tau^{ij} = \left(h_i, \frac{1}{1 + e^2 \cdot G_\beta} h_j\right), \quad i, j = 1, 2, \ldots, n. \tag{36}$$

Here h_j are the indicator functions of time intervals $0 \leqslant s \leqslant t_j$, $j = 1, 2, \ldots, n$ in $\overline{\mathbb{R}}_+$ and k stands shortly for the sum of all k_j's. The diagonal elements of the matrix τ^{ij} yield a modified function $t \to \tau(t)$, $t \geqslant 0$, satisfying

$$\tau(s + t) = \tau(s) + \tau(t) - \frac{e^2}{\gamma} \cdot \delta(s)\, \delta(t). \tag{37}$$

Let us introduce $L^2(Q)$ with scalar product given by the measure $d\mu$ and denote R the projection operator onto the subspace $L^2(Q_+)$ of functions which are measurable with respect to Σ_+.

Then [24]

$$\mathscr{K} = \overline{L^2(Q_+)/\ker W}, \tag{38}$$

where $W = +(R\Theta R)^{1/2}$, can be identified with the physical Hilber space. Since $d\mu$ violates the Markov property $\mathscr{K}$ is larger than $L^2(S)$. Indeed we find an isometry $J: \mathscr{K} \to \mathscr{H}$ so that

$$\begin{aligned} \Psi(t) &= JWT^t\, e^{ikx} \otimes 1 \\ &= c(t, k^2) \cdot \exp\{ik(x + \gamma^{-2} \cdot e\delta(t)E)\}\Omega, \end{aligned} \tag{39}$$

where E was the canonical conjugate to A. For details see [25]. On the classical level the elimination of the "unvisible" oscillator leads to the following integro-differential equation

$$m\ddot{x}(t) = -e^2\left\{x(t) + y \cdot \int_{t_1}^{t_2} C_\beta(s, t)x(s)\,ds\right\}. \tag{40}$$

Above $-\beta^2 C_\beta(\cdot, \cdot)$ stands for the periodic Green's function of the hyperbolic operator $\beta^2 + d^2/dt^2$ on (t_1, t_2) [26]. Of course one may choose other boundary conditions as well.

Acknowledgements

I would like to thank S. Albeverio and R. Gielerak for recalling may attention to Yngvason's result at XXIV. Karpacz Winterschool 1988 in Poland. Discussions with A. Jadvczyk, E. Seiler, B. Crell, K. Kürsten, J. Friedrich, K. Schmüdgen, H. Grosse, L. Landau, R. Seiler helped me to complete the work. Partly it was performed at JINR Dubna, INRNE Sofia but also at Conference in Poiana Braşov 1989 some ideas were born.

REFERENCES

[1] R. P. Feynman and A. R. Hinbbs, *QM and Path Integrals*, McGraw Hill, N.Y. (1965).
[2] K. Osterwalder and R. Schrader, *Comm. Math. Phys. 31*, 83 (1973).
[3] H.-J. Borchers, *Proc. of XV. Karpacz Winter School*, Poland, 1978, *Acta Wratisl.* No. *519.*
[4] J. Yngvason, *Rep. on Math. Physics 13*, 101 (1978).
[5] D. Applebaum, *Fermion stochastic calculus*, Univ. of Nottingham Ph.D. Thesis (1984).
[6] K. D. Rothe and J. A. Świeca, *Nucl. Phys. B138*, 26 (1978).
[7] Yu. L. Daletsky/S. V. Fomin, *Measures in ∞-dim. spaces* (Russian: Ed. Nauka, Moscow 1983).
[8] A. O. Caldeira and A. J. Leggett, *Ann. Phys. 149*, 374 (1983).
[9] E. Nelson, in: *Lect. Notes in Physics 25*, 105 Springer-Verlag (1973).
[10] G. Arfken, *Math. Methods for Physicists* (Russian: Ed. Nauka, Moscow 1970).
[11] J. Feldman, *Probability theory and its applications X*, 375 (Academy of Sciences USSR, Ed. Nauka, Moscow 1965).
[12] D. Mumford, *Tata Lect. on theta I*, in: *Progress in Math. Vol. 28*, Birkhäuser-Verlag (1983).
[13] J. L. Challifour, XV. Schladming Univ.-wochen (1975).
[14] F. Guerra, L. Rosen and B. Simon, *Ann. Math. 101*, 111 (1975) and: J. Fröhlich, *Ann. of Physics 97*, 54 (1976).
[15] T. Hida, J. Potthoff and L. Streit, *On quantum theory in terms of white noise*, *Nagoya Math. J. 68*, 21 (1977).
[16] T. Kato, *Perturb. Theory for Linear Operators*, Springer-Verlag (1966).
[17] E. Seiler, *Lect. N. Phys. 159*, Springer (1982).
[18] H. Flanders, *Diff. Forms and Application to Physics*, Acad. Press (1983).
[19] M. C. Reed, *Functional analysis and probability theory*, in *Lect. N. Phys. 25*, Springer-Verlag (1973).
[20] *Quotation of A. Einstein (due to L. Infeld)*, in: *Einstein wider Denkgewohnheiten*, WTB Akademie-Verlag (1976).
[21] B. Simon, *Helv. Phys. Acta 46*, 686 (1973).
[22] R. Høegh-Krohn, *Comm. Math. Phys. 38*, 195 (1974).
[23] A. Klein, *Bull. Americam Math. Soc. 82*, 762 (1976).
[24] G. C. Hegerfeldt, *Comm. Math. Phys. 35*, 155 (1974).
[25] J. Löffelholz, *Ann. d. Physik*, Bd. *46* Heft 1, 55 (1989), and Preprint Universität Leipzig KMU/NTZ *08* (1990).
[26] D. C. Khandekar, S. V. Lawande and K. V. Bhagwat, *J. Phys. A16*, 4209–4220 (1983).