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Autor: Yilmazer, A. Ulvi / Aydin, Z. Zekeriya
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Lorentz invariance from Euclidean supersymmetry

By A. Ulvi Yilmazer and Z. Zekeriya Aydin

Department of Engineering Physics, University of Ankara, 06100 Beşevler, Ankara, Turkey

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Abstract. The Schrödinger equation in a super-Euclidean space is examined from the Lagrangian field theory point of view. It is explicitly shown that the elimination of the "auxiliary" field makes the Lorentz invariance manifest, but destroys the manifest super-symmetry. This suggests that it could be sufficient to supersymmetrize the relativistic systems in Euclidean level.

In a recent paper [1] Sokatchev and Stoyanov have written an equation of Schrödinger type for a chiral super-field using the supersymmetric extension of the three dimensional Euclidean symmetry. It is interesting to observe that although this supersymmetric equation possesses only Euclidean invariance, the equations of motion for the physical components of the superfield, after the elimination of the auxiliary field, turn out to be Lorentz invariant. Also it is shown later that [2, 3] the principle of minimal coupling to the electromagnetic field can be adopted to obtain the relativistic wave equations for particles interacting with external fields, and this model is extended to two-body systems.

Here we present a better understanding of the model from the Lagrangian field theory point of view. As is pointed above the supersymmetry algebra in this approach is the super-extension of the three dimensional Euclidean group. The algebra satisfied by the generators of this group can be written as

$$\begin{aligned} &[J_k, J_l] = i\epsilon_{klm} J_m \qquad &&[J_k, Q_\alpha] = -\tfrac{1}{2}\sigma^k_{\alpha\beta} Q_\beta \\ &[J_k, P_l] = i\epsilon_{klm} P_m \qquad &&[P_k, Q_\alpha] = 0 \\ &[P_k, P_l] = 0 \qquad &&\{Q_\alpha, Q_\beta\} = 4(\sigma^l \epsilon)_{\alpha\beta} P_l \end{aligned} \tag{1}$$

where P_k's and J_k's ($k = 1, 2, 3$) are the translations and 0(3) generators respectively. The supersymmetry generator Q_α is a Weyl spinor. In the superspace (t, x_k, θ_α) a supersymmetric equation in the Schrödinger form is

$$i\frac{\partial}{\partial t}\phi(t; \vec{x}, \theta) = \tfrac{1}{4} D^\alpha D_\alpha \phi(t; \vec{x}, \theta) \tag{2}$$

where D_α is the covariant derivative given as

$$D_\alpha = i\frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^k\theta)_\alpha P_k \tag{3}$$

and $\phi(t; \vec{x}, \theta)$ is the chiral superfield which has the usual θ-expansion:

$$\phi(t; \vec{x}, \theta) = A(t, \vec{x}) + \theta^\alpha \psi_\alpha(t, \vec{x}) + \theta^\alpha \theta_\alpha B(t, \vec{x}) \tag{4}$$

After the elimination of the auxiliary field B the equation (2) leads to the Klein–Gordon and Dirac equation for the scalar field A and spinor field ψ respectively [1].

Now we wish to construct the action invariant under the Euclidean supersymmetry transformations such that it gives rise to the Eq. (2) as a field equation in superspace. In order to choose the simplest and natural one among the several possible candidates we recall the well-known fact that the ordinary non-relativistic Schrödinger equation $-1/2m\vec{\nabla}^2\psi + V\psi = i\dot{\psi}$ for a particle of mass m in a potential $V(x)$ may be regarded as a field equation and derived from the following Lagrangian density:

$$\mathscr{L}_s = -\frac{1}{2m}\vec{\nabla}\psi^* \cdot \vec{\nabla}\psi + i\psi^*\dot{\psi} - V\psi^*\psi \tag{5}$$

Thus with this analogy we propose the following Lagrangian density

$$\mathscr{L} = \int d^2\theta \, [\tfrac{1}{4}(D\phi^+)(D\phi) + i\phi^+\dot{\phi}] \tag{6}$$

which is invariant under Euclidean supersymmetry transformations $t' = t$, $x'_k = x_k + \theta\sigma_k\bar{\xi}$, $\theta'_\alpha = \theta_\alpha + \xi_\alpha$, $\delta\phi = (\xi Q)\phi$.

We note that the Euler–Lagrange field equation

$$\frac{\partial\mathscr{L}}{\partial\phi} - \frac{\partial}{\partial t}\left(\frac{\partial\mathscr{L}}{\partial(\partial\phi/\partial t)}\right) - \frac{\partial}{\partial\theta}\left(\frac{\partial\mathscr{L}}{\partial(\partial\phi/\partial\theta)}\right) - \frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial\mathscr{L}}{\partial(\partial\phi/\partial\bar{\theta})}\right) = 0 \tag{7}$$

can be written as

$$\frac{\partial\mathscr{L}}{\partial\phi} = \frac{\partial}{\partial t}\frac{\partial\mathscr{L}}{\partial(\partial\phi/\partial t)} + D\frac{\partial\mathscr{L}}{\partial(D\phi)} + \bar{D}\frac{\partial\mathscr{L}}{\partial(\bar{D}\phi)} \tag{8}$$

because the Lagrangian (6) is in the form $\mathscr{L} = \mathscr{L}(\phi, \phi^+, \dot{\phi}, \dot{\phi}^+, D\phi, D\phi^+)$. Then the supersymmetric equation (2) can easily be derived from the above Lagrangian density using (8).

On the other hand the Lagrangian density $\mathscr{L}$ can explicitly be written in terms of the superfield components as

$$\mathscr{L} = \int d^2\theta \, [-i\bar{\psi}^\alpha(\sigma^k\theta)_\alpha\theta^\beta(\partial_k\psi_\beta) - (\sigma^k\theta)^\alpha(\sigma^l\theta)_\alpha(\partial_k\bar{A})(\partial_l A)$$
$$- \theta^\alpha\theta_\alpha\bar{B}B + i\theta^\alpha\theta_\alpha(\dot{B}\bar{A} + \bar{B}\dot{A} + \bar{\psi}\dot{\psi})] \tag{9}$$

where we have retained only the $\theta\theta$-terms due to the $d^2\theta$ integration and used the relation $\theta\sigma^k\theta = 0$. We see that the equation of motion for the auxiliary field B is

$$B = i\dot{A} \tag{10}$$

and consequently it can be eliminated from the Lagrangian density. The resulting Lagrangian is hence

$$\mathscr{L} = -i\bar{\psi}\sigma^{\mu}\,\partial_{\mu}\psi + (\partial^{\mu}\bar{A})(\partial_{\mu}A) \tag{11}$$

which is the sum of the Lagrangian densities for the massless Dirac and Klein–Gordon fields. In order to introduce masses one should add usual $1/2\int d^2\theta m\phi\phi^{+} +$ herm.conj. term to the supersymmetric Lagrangian (6) to that the equation of motion for the superfield ϕ becomes

$$i\dot{\phi} = \tfrac{1}{4}D^{\alpha}D_{\alpha}\phi - m\phi^{+} \tag{12}$$

It is easy to show that the above equation leads to the massive Dirac and Klein–Gordon equations after the elimination of the auxiliary field B.

The extension of the above idea to the Maxwell as well as Yang–Mills field theories can also be worked out. To construct three dimensional Euclidean supersymmetric YM theory one needs a spinor gauge connection superfield $X_{\alpha}(t, \vec{x}, \theta)$ and the corresponding field strength $W_{\alpha} = D^{\beta}D_{\alpha}X_{\beta}$. This model has been studied by Aneva [4] where it is postulated that gauge invariant superfield W_{α} satisfies a Schrödinger type equation similar to Eq. (2). Then the equations for the spinor and vector components of the superfield W_{α} are shown to be the Weyl, and Maxwell equations of electrodynamics respectively, but without any need for the auxiliary fields in the model.

Discussion

As is well known auxiliary fields have many uses in supersymmetric field theories. For example they allow the supersymmetry to be realized off shell, so actions are easier to construct. They have trivial (non-dynamical) equations of motion and can be eliminated from the action without loss of generality. In our case we see that some subtlety exists in this context. In Euclidean superspace the fields A and B are complex scalars and ψ is a SU(2) spinor. However we see from the Eq. (10) that the auxiliary field is not a Lorentz scalar and one can not say that it is completely non-dynamical, because the elimination of B makes the equation of motion for the physical field A relativistic. The field B can be considered to be an auxiliary one in the very standard sense of this term, because its equation of motion does not involve derivatives, so it is just a Lagrange multiplier. We observe that the supersymmetric Lagrangian (6) has two terms which are not Lorentz invariant separately. However the elimination of the field B through its field equation makes the Lorentz invariance manifest but destroys the manifest supersymmetry.

Finally this formalism suggests that in order to construct covariant and supersymmetric actions one can start from a lower symmetry, namely Euclidean superspace. Supersymmetric field theories are usually established by starting with Super-Poincaré algebra. In these models the elimination of the auxiliary fields produce the scalar mass terms as well as the scalar potential, and although the manifest supersymmetry is destroyed during this process the Lorentz invariance remains to be valid all the time. In our case the elimination of the auxiliary field changes the manifest symmetries of the model.

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